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First-order shear deformation theory for orthotropic doubly-curved shells based on a modified couple stress elasticity

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A B S T R A C T

This paper investigates the micro- and nano-mechanical behavior of orthotropic doubly-curved shells by considering the New Modified Couple Stress Theory (NMCST). The higher order continuum assumed by the NMCST includes three material length scale parameters in order to capture the size-effect of anisotropic and orthotropic materials. The governing equations of the problem are based on the First-order Shear Deformation Theory (FSDT). According to the proposed NMCST, the expressions of the physical components for the strain and curvature tensors are obtained in an orthogonal curvilinear coordinate system. Then, the governing differential equations and boundary conditions are derived by applying the energy method and Hamilton's principle. A comparative investigation between our numerical results and the ones available in the literature proves the capability of the proposed formulation in predicting the micro- and nano-mechanical behavior of orthotropic doubly-curved shells.

1. Introduction

Due to the incapability of classical theories to investigate the effects of length-scale parameters on the deformations caused by static and dynamic loads in Nano Electro Mechanic Systems (NEMS), Micro Electro Mechanic Systems (MEMS), actuators, atomic microscopes, and so on, researchers have widely addressed the higher-order continuum theories, e.g. the Couple Stress Theory (CST) [1–3], the Modified Couple Stress Theory (MCST) [4–16], the New Modified Couple Stress Theory (NMCST) [17–20], the Strain Gradient Theory (SGT) [21–28], the Modified Strain Gradient Theory (MSGT) [21,29–32], the Eringen's Nonlocal Theory (ENT) [33–37], among others. The main difference between the classical theories and the higher-order ones can be found in the definition of stress–strain relations. Among these theories, the MCST was first presented by Yang et al. [38] based on the CST. Starting from this pioneering work, many other studies employed the CST, MCST, and NMCST to analyze the mechanical phenomena for different engineering applications. For instance, the MCST was developed by Dehrouyeh-Semnani et al. [39] for investigating the dynamic behavior of axially moving micro-beams and its size-dependence. Based on NMCST, Chen et al. [17] proposed a composite laminated Reddy's plate model. The NMCST was used by Chen et al. [18] to analyze the free vibration at the micro-scale for laminated composites treated with the Timoshenko beam model. Chen et al. [40] applied the NMCST to study the anisotropic elasticity and micro-scale behavior of laminated Kirchhoff plate models. Further examples related to the application of the Couple Stress Theory can be found in the papers [41–53].

Shell-type constructions are widely used in many branches of industry (e.g. shipbuilding, aircraft structures, turbine disks, etc...) owing to their light weight, tailor-made and excellent properties in terms of dynamic behavior, strength and stability [54–58]. For these reasons, many researchers have increasingly dedicated their efforts to develop shell theories. More in detail, the double curvature of a shell makes it more difficult to analyze [59]. The first studies on the elastic behavior of doubly-curved shells can be found in the 1940s [60–65], and many other studies and investigations have been increasingly performed in the recent literature. Tornabene et al. [60] applied

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the higher-order Layer-Wise (LW) theories to analyze doubly-curved laminated shells and panels with soft-core characteristics. Based on higher-order structural theories, Tornabene et al. [66] investigated the static analysis of doubly-curved composites panels strengthened by curvilinear fibers. Shooshtary et al. [67] studied linear and nonlinear free vibration of symmetrically laminated magneto-electro-elastic doubly curved shell resting on elastic foundation. Viola et al. [68] employed higher-order shear deformation theories to investigate the static behavior of completely doubly-curved shells and panels. Veysi et al. [69] studied the nonlinear vibrations of micro doubly-curved shallow shells by taking into account the MCST. The doubly-curved shells are also straightforwardly analyzed by Amabili [70–74], Amabili and Reddy [75], Tornabene [76–78], among others.

Despite the large amount of researches conducted so far on doubly-curved shells, there is a lack of information about the efficiency of the NMCST when combined with the First-order Shear Deformation Theory (FSDT) to study the mechanical behavior of orthotropic doubly-curved shells. The paper is organized in the following manner. The NMCST is first reviewed in Section 2. The geometrical description of shell is provided in Section 3, whereas the governing equations of the problem and boundary conditions are obtained in Section 4 based on the application of the Hamilton's principle. Section 5 is concerned with the numerical modeling and application for cylindrical shells, whereas the main results are comparatively evaluated and discussed in Section 6. Finally, conclusions are drawn in Section 7.

2. New modified couple stress theory

The classical couple stress theory includes two parameters to describe the effect of the material length scale. Yang et al. [38] proposed a MCST in which the two material length scale parameters reduce to one parameter for the study of micro and nano-scale structures. MCST is generally adopted in the literature for isotropic materials. Chen et al. [18] presented a NMCST based on three length scale parameters for treating anisotropic and orthotropic materials. According to the NMCST, the strain energy in Cartesian coordinates can be defined as

$$U = \frac{1}{2}\int (\sigma : \varepsilon + m : \chi) dV \tag{1}$$

with

$$\begin{cases} \sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \\ m_{ij} = (\ell_i^2 G_i \chi_{ij} + \ell_j^2 G_j \chi_{ji}) \quad (i, j = 1, 2, 3), \end{cases} \tag{2}$$

and

$$\begin{cases} \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \\ \chi_{ij} = \omega_{i,j}, \end{cases} \tag{3}$$

In the above relations, ℓ_i refer to the material length scale parameters, C_{ijkl} and G_j are the elasticity constants, σ_{ij} and ε_{ij} are the stress and strain tensors, respectively, χ_{ij} is the curvature tensor, and m_{ij} is the new modified couple stress tensor.

The strain tensor ε_{ij} and the curvature tensor χ_{ij} can be redefined in curvilinear coordinates as follows (see Ashoori et al. [79] and Zhao et al. [80]).

$$\varepsilon_{ij} = \frac{1}{2}(u_{i|j} + u_{j|i}) \tag{4}$$

$$\chi_{ij} = \theta_{j|i} = \frac{1}{2} g_{jk}\Im^{klm}\eta_i \ell_m = \frac{1}{2} g_{jk}\Im^{klm} u_{m|li} = \frac{1}{2} g_{jk}\Im^{klm}\eta_{ilm} \tag{5}$$

where the symbol $|_{(\,)}$ refers to the covariant derivative defined for a typical third rank tensor A^i_{jk} by

$$A^i_{jk|_m} = \frac{\partial A^i_{jk}}{\partial x^m} + A^l_{jk}\Gamma^i_{lm} - A^i_{lk}\Gamma^l_{jm} - A^i_{jl}\Gamma^l_{km} \tag{6}$$

In Eq. (6) x^i $(i = 1, 2, 3)$ is the curvilinear coordinate in Euclidean space, and g_{ij} refer to the covariant components of the Euclidean metric tensor.

The second kind of Christoffel symbol is defined by

$$\Gamma^i_{jk} = \frac{1}{2} g^{il}\left(\frac{\partial g_{jl}}{\partial x^k} + \frac{\partial g_{kl}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l}\right) \tag{7}$$

where g^{kl} are the contravariant components of the Euclidean metric tensor, $\Im^{ijk}$ and η_{ijk} are the contravariant Levi-Civita tensor and the second order deformation gradient, respectively, which can be expressed as

$$\eta_{ijk} = u_{k|ij} \tag{8}$$

$$\Im^{ijk} = \frac{1}{\sqrt{g}} e^{ijk} \tag{9}$$

In orthogonal curvilinear coordinates, covariant and contravariant components of the Euclidean metric tensors are computed as shown below

$$g^{\underline{ii}} = \frac{1}{g_{\underline{ii}}}, \qquad g = \det g_{ij} = g_{11}g_{22}g_{33} \tag{10}$$

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