# A biobjective branch and bound procedure for planning spatial missions 

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## A R T I C L E I N F O

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#### Abstract

More than $90 \%$ of the space objects orbiting around the earth are space debris. Since the orbits of these debris often overlap the trajectories of spacecraft, they create a potential collision risk. The problem of removing the most dangerous space debris can be modeled as a biobjective time dependent traveling salesman problem (BiTDTSP). In this paper, we study an approach based on a branch and bound procedure to determine the Pareto frontier of the BiTDTSP.


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## 1. Introduction

Since the launch of the first satellite in 1957, an increasing number of objects has been put into orbit around the earth. The number of space objects with diameter above 10 cm is estimated, nowadays, at about 15,000 . Only $10 \%$ of these objects are operational, the other objects constitute space debris. Even if future launches are suspended, the number of space debris will continue to increase, e.g. owing to collisions between debris, making the collision with an operational satellite more probable [10]. Since consequences of collisions with debris may prove dramatic, avoidance maneuvers or missions to remove debris are necessary [5]. As removing all space debris would be quite expensive, the idea is to determine the debris which are more likely to cause collisions and remove them at least cost. Removal is performed by achieving a space rendezvous between a moving space vehicle and each debris followed by a soft capture using a robotic arm. The shuttle has to meet each debris on its orbit until all debris have been dealt with and then return to its initial orbit. This tour has to be achieved in the less expensive and the fastest possible way. Most collisions are not debris/satellite but rather debris/debris and result in an in-

[^0]creasing number of space debris [9,1]. So the earlier the removal is finished, the less new debris are generated.

In this article, we study the problem of removing a list of space debris and extend the results obtained in [12]. We propose an exact algorithm based on a branch and bound procedure to compute the set of non-dominated (cost, duration) vectors and give for each of these vectors a feasible solution. Two specificities that make our model more realistic are taken into account: between each pair of targets, several transfer possibilities, with different costs and times, are considered. These costs and times depend on the start time from the initial target in the pair. Therefore, the problem is modeled as a biobjective time dependent traveling salesman problem defined on a multigraph. To the best of our knowledge, BiTDTSP has not been studied before, although several articles study the time dependent traveling salesman problem (TDTSP) and the biobjective traveling salesman problem (BiTSP). TDTSP is a variant of TSP where distances depend on the arrival time to each vertex. Malandraki and Daskin [13] described dynamic programming (DP) algorithms for TDTSP extended by Bellman [3] and finally Held and Karp [7]. Schneider [17] also proposed a simulated annealing heuristic to deal with TDTSP. BiTSP has been addressed by several authors. Gendreau et al. [4] used the $\epsilon$-constraint method to efficiently generate the Pareto front of the traveling salesman problem with profits. Schmitz and Niemann [16] were interested in a BiTSP problem motivated by various applications in the context of service delivery in which the second objective relates to priorities among locations to be visited. Paquette and Stutzle [14] analyzed algorithmic components of Stochastic local search (SLS) algorithms for the


Fig. 1. The transfer orbit of the space shuttle using the elliptical maneuver.
multiobjective traveling salesman problem. Based on the insights gained, they engineered SLS algorithms for this problem. Lust and Jaszkiewicz [11] proposed a heuristic resolution based on the twophase local search method with speed-up techniques for BiTSP.

The paper is organized as follows. In Section 2, we model the problem of removing dangerous space debris. The proposed approach is presented in Section 3. Section 4 is devoted to implementation issues. Computational experiments and results are reported in Section 5 and conclusions are provided in a final section.

## 2. Problem description

### 2.1. Context and notations

Given $n$ debris to be removed, the space shuttle has to move from its own orbit and visit the $n$ debris in order to collect them and then return to its first orbit. The total quantity of fuel burned during each transfer from a debris $i$ to a debris $j$, denoted by $c_{i j}$, represents the transfer cost. The duration of the transfer is denoted by $d_{i j}$. The quantity of fuel burned during the mission should not exceed the shuttle capacity, thus the cost cannot exceed a fixed cost $c_{\text {max }}$. Moreover, the duration of the mission should not exceed a fixed duration $d_{\max }$. We assume here that the mission is not carried out by an unmanned space shuttle. To remove each debris, the shuttle has to perform a rendezvous with each debris on its orbit. Thus in the following we associate each debris with its orbit. Costs and durations depend on the way the rendezvous is achieved.

A space rendezvous between a debris and a space shuttle is an orbital maneuver where both arrive at the same orbit and approach to a very close distance. There are several ways to achieve a space rendezvous as shown in [6,19,18]. In our case, we have chosen to perform a rendezvous following the Lambert method where the shuttle moves between the two orbits undergoing exactly two pulses. Fig. 1 shows how the transfer is performed in the Lambert elliptical case. The cost of this transfer is the quantity of fuel burned in order to perform the first and second pulses.

For each debris $i, t_{i}$ denotes the time at which the shuttle reaches the orbit of debris $i$. Once the shuttle has reached orbit $i$, it can immediately start the next transfer to reach another orbit $j$ or wait before beginning the transfer. Indeed, waiting on an orbit may be cheaper and/or quicker to reach the next orbit. The duration of the transfer is the sum of the waiting time in the departure orbit and the travel time to the arrival orbit. In the following we assume that each elementary transfer requires a minimum duration $\delta t_{\text {trans }}$ and a minimum cost $\delta c_{m i n}$. We assume as well that the service time on an orbit $i$ takes a duration of $\delta t_{\text {serv }_{i}}$.

### 2.2. Formulation of the problem

The studied space objects and the possible transfers between their orbits are modeled by a complete valued digraph $G=(V, A)$ where $V$ represents the set of object's orbits numbered from 0 to $n$. Vertex 0 represents the shuttle and vertices from 1 to $n$ represent the $n$ debris. The set $A$ corresponds to the set of feasible transfers between orbits. Several arcs may link each pair of orbits depending on the moment on when the shuttle reaches the departure orbit. To each 3-tuple ( $i, j, t_{i}$ ), corresponds a set of feasible transfers $A_{i j}\left(t_{i}\right)$. Each element of $A_{i j}\left(t_{i}\right)$ induces a bivalued arc linking $i$ to $j$ whose value is a pair ( $c_{i j}, d_{i j}$ ) corresponding to the cost and duration of the travel. As the arcs representing possible transfers depend on the time at which the shuttle reaches the departure orbit, $G$ is a dynamic multigraph. In the Fig. 2, we have shown the way the digraph is constituted as the shuttle performs its mission.

After it reaches an orbit $i$, the shuttle must serve it before going on. Thus, the shuttle can leave the orbit $i$ at least at $t_{i_{d}}=$ $t_{i}+\delta t_{\text {serv }_{i}}$. Due to mission duration constraints, the shuttle should leave the orbit $i$ before an instant limit $t_{i_{l}}$. When the shuttle leaves $i$ at $t_{i_{l}}$, it has barely time to achieve the mission before $t_{0}+d_{\max }$. Thus, if the shuttle reaches $l$ after visiting $V^{\prime} \subset V$ debris, one has
$t_{i_{l}}=t_{0}+d_{\max }-\left(n+1-\left|V^{\prime}\right|\right) \times \delta t_{\text {trans }}-\sum_{i \in V \backslash V^{\prime}} \delta t_{\text {serv }_{i}}-\delta t_{\text {trans }}$
In the equation above, $\left(n+1-\left|V^{\prime}\right|\right) \times \delta t_{\text {trans }}$ is the least time needed to travel to each unvisited debris, $\sum_{i \in V \backslash V^{\prime}} \delta t_{\text {serv }}$ is the time needed to serve debris and $\delta t_{\text {trans }}$ is required to go back to the first orbit. The duration of time corresponding to possible departure times is discretized using a time step $\delta t$. Hence, the transfer can start at any time $t_{i_{d}}+w \times \delta t$, where $w \in I\left(t_{i}\right)$ and $I\left(t_{i}\right)$ determines the set of possible departure times if the shuttle reaches orbit $i$ at $t_{i}$ that is $I\left(t_{i}\right)=\left\{0,1, \ldots,\left\lfloor\frac{t_{i}-t_{i d}}{\delta t}\right\rfloor\right\}$.

For each departure time several transfer durations are possible, the set of possible durations is denoted as $D$. A departure time $t_{i}$, $w \in I\left(t_{i}\right)$ and transfer duration $p \in D$ define a new arc allowing the shuttle to reach $j$ at $t_{j}=t_{i_{d}}+d_{i j}\left(t_{i}, w, p\right)$. The valuation of this arc is:
$\left(c_{i j}\left(t_{i}, w, p\right), d_{i j}\left(t_{i}, w, p\right)\right)$.
The transfer possibilities can be seen in Fig. 3.

## 3. Solution approach

### 3.1. Preprocessing

The cardinality of $A_{i j}\left(t_{i}\right)$ depends on the number of possible departure times from $i$ and on the number of possible durations to perform the transfer from $i$ to $j$. Therefore $A_{i j}\left(t_{i}\right)$ may contain a very large number of possible transfers. The computation of the cost corresponding to each transfer possibility using Lambert algorithm is very time-demanding (see Section 5.2.2). The algorithm spends much more time computing transfer costs than optimizing the shuttle trajectory. Therefore, optimizing the algorithm performances requires limiting the number of calls to the cost computation function. This is achieved by reducing the cardinality of $A_{i j}\left(t_{i}\right)$ as will be seen in Section 5.2.1.

### 3.2. The branch and bound procedure

In order to enumerate the non-dominated vectors for BiTDTSP, we propose a branch and bound enumeration scheme. In our case,

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