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Stability Analysis of a Multirotor Vehicle Hovering Condition

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Abstract

This paper presents a unified framework for addressing the stability analysis of the hovering condition for a multirotor vehicle. By linearization about a trim condition, a decoupled model made of four dynamic modes is derived. A set of stability derivatives is obtained and the effect on natural dynamic stability of design parameters such as arm dihedral and rotor tilt angles is analyzed. A numerical test case is finally discussed in support of the proposed approach.

1. Introduction

The interest in multirotor Remotely Piloted Aerial Systems (RPAS) for research and civil applications showed a steep increase in the last decade, along with a growing availability of miniaturized avionic systems, novel design tools for optimal performance [1, 2], efficient control laws [3, 4], and accurate estimation algorithms [5, 6].

In order to properly address the design of novel multirotor configurations and stabilization control laws, a deep comprehension of the aircraft natural dynamics is required. Many off-the-shelf multirotor platforms as the DJI® S800 and S1000 are provided with a fixed non-planar displacement of rotor disks, such that the thrust generated at hover by the individual rotor is inclined with respect to the local vertical. It is the case when a 'dihedral' angle is provided to each rotor arm and a 'tilt' angle deviates the rotor thrust from the vertical plane that contains the corresponding rotor arm. It is common knowledge that such design solutions may provide some kind of passive stability, allowing the vehicle to be re-leveled at hover after attitude perturbations [7]. As a matter of fact, a comprehensive analysis for the engineering use of such instruments is lacking, which is reflected by design and sizing procedures that are still entrusted to experience developed in the world of model-making. Although the behavior of an isolated rotor was widely studied in the past years, few results are in fact available in the literature about the interaction of two or more rotors. In [8] it is demonstrated that the stability of the longitudinal and lateral motion of a tandem-rotor helicopter largely depends on small differences between the thrust vectors of the front and rear rotor. In particular, it is proven that, in order to eliminate divergence in the longitudinal dynamic stability, a suitable value of swash-plate dihedral angle is necessary between the two rotors. In [9] the influence of the rotor dynamics is also explored for a multirotor configuration and the necessity to calculate the rotor thrust derivatives far more accurately than for the single-rotor helicopter is emphasized.

This paper presents an analytical study of the open-loop dynamics for a multirotor vehicle in the hovering flight condition. Starting from the nonlinear rigid-body equations of motion, a linearized model made of four decoupled subsystems is obtained and approximate closed-form solutions for dynamic modes are derived as a function of a set of stability derivatives. In particular, a detailed analysis of rotor thrust derivatives proves how design parameters such as dihedral and tilt angles may actually affect the stability of the hovering condition. Although many publications deal with stability analysis of rotor systems by using advanced modeling tools (typically multibody), which incorporate sophisticated sub-models for the aero-servo-structural dynamics [10], it is in the author's intention to provide an analytical framework in which the natural dynamics of a rotorcraft is clearly related to the configuration parameters. In this respect, the simplicity of the approach represents a valuable aspect when the design process or the synthesis of control laws is at a preliminary stage.

2. Multirotor nonlinear model

A multirotor platform is considered where four identical motors drive fixed-pitch propellers with paired spin directions. Extension to $N > 2$ pairs of rotors is straightforward. Without loss of generality, it is assumed that rotor arms lie on the same plane while the thrust vector generated by the individual rotor disk is inclined by fixed dihedral and tilt angles. A sketch of the vehicle and the main reference frames is shown in Fig. 1.

2.1. Reference frames

The following reference frames are introduced:

1. An Earth-fixed North-East-Down frame, $\mathbb{F}_E = \{O; \boldsymbol{x}_E, \boldsymbol{y}_E, z_E\}$. This frame is inertial under the assumption of flat and non-rotating Earth.
2. A Body-fixed frame, $\mathbb{F}_B = \{P; \boldsymbol{x}_B, \boldsymbol{y}_B, z_B\}$, that defines the vehicle principal axes of inertia and whose origin, P, is located at the multirotor center of gravity (c.g.).

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