Reliability Engineering and System Safety 96 (2011) 619–626

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

# Stochastic analysis of shock process and modeling of condition-based maintenance

J.A.M. van der Weide [a,*], M.D. Pandey [b]

[a] Delft University of Technology, Delft, The Netherlands
[b] University of Waterloo, Waterloo, Canada

ARTICLE INFO

Available online 11 January 2011

*Keywords:*
Shock processes
Condition-based maintenance
Non-homogeneous Poisson process
Reliability
Renewal theorem
Stochastic degradation

ABSTRACT

The paper presents an analytical formulation for evaluating the maintenance cost of engineering systems that are damaged by shocks arriving randomly in time. The damage process is nonlinear in a sense that damage increments form an increasing sequence (i.e., accelerated damage) or a decreasing sequence (saturated damage) of random increments. Such processes are motivated from damage data collected from nuclear reactor components. To model the nonlinear nature of damage process, the paper proposes the use of non-homogeneous Poisson process for damage increments, which is in contrast with the common use of a renewal process for modeling the damage. The paper presents a conceptually clear and comprehensive derivation of formulas for computing the expected cost rate associated with a periodic inspection and preventive maintenance policy. Distinctions between the analysis of self-announced and latent failures are highlighted. The analytical model presented in this paper is quite generic and versatile, and it can be applied to optimize other types of maintenance policies.



## 1. Introduction

### 1.1. Motivation

Critical engineering systems, components and structures in power plants, chemical processing industry, and automotive industry are vulnerable to failure due to damage caused by shocks (or over-stress) that occur over the service life of the system. The failure occurs when the damage in the system exceeds its capacity. To ensure reliability of such systems, inspection and preventive maintenance programs are implemented by engineers.

This paper presents a probabilistic analysis of the cost of a maintenance program that includes periodic inspection and condition-based preventive maintenance. The system is subjected to shocks and the damage is modeled as a cumulative stochastic process. A cumulative damage models has two stochastic components, which in the current literature are modeled as follows. The random occurrence of shocks in time is typically modeled as the Poisson or the renewal process. The damage increments caused by shocks are modeled as *iid* random variables. It can be interpreted that in the space of damage, the damage increments constitute a renewal process. Thus, the damage increments are uncorrelated. This model is generally referred to as the compound renewal process [1]. Note that damage increments are analogous to the gap length in the point process.

The modeling of damage increments as a realization of renewal process is not suitable to model nonlinear damage processes encountered in some nuclear plant systems. This paper is considered with two particular types of nonlinear behavior. The first is the "accelerated" damage model, in which the damage increments are increasing with the accumulation of damage. A practical example of this case is damage in rotating machinery (such as a motor) due to an unbalanced shaft. The unbalanced shaft increases the vibration amplitude causing damage to the shaft and the motor. The damaged shaft accentuates the vibrations, which further increases the damage, and in this way nonlinearity are introduced. The second type of nonlinearity is seen in a "saturated" damage process, in which the damage increments are decreasing with the accumulation of damage. The radiation induced damage to the material of reactor components is this type of damage. The radiation process generates dislocation defects, but these defects cannot increase indefinitely. So damage increments follow a decreasing sequence, implying that damage is reaching a saturation limit.

Since the classical compound renewal process is not apt to modeling such nonlinear damage processes, the central objective of the paper is to develop a more general shock model in which the assumption of independent damage increments is relaxed by modeling them as a realization of non-homogeneous Poisson

* Corresponding author.
*E-mail address:* j.a.m.vanderweide@tudelft.nl (J.A.M. van der Weide).

0951-8320/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ress.2010.12.012

process (NHPP). An accelerated damage processes can be emulated by an NHPP with an increasing intensity function, which will imply decreasing damage increments (or gap length) with the accumulation of damage. A saturated damage process can be modeled as an NHPP with decreasing intensity function, such that the damage increments will form an increasing sequence with the accumulation of damage. The proposed model also includes finite repair time and explains distinctions between the analysis of self-announced versus latent failures.

### 1.2. Background literature

An extensive review of literature and in-depth analysis of cumulative stochastic damage model have been presented in a number of recent monographs, such as Manzini et al. [2], Nakagawa [3–5], Wang and Pham [6], Tijms [7] and Aven and Jensen [8]. Therefore, this paper avoids a long literature review for the sake of not being repetitive.

Manzini et al. [2, Section 10.4, p. 402] discussed a condition-based preventive maintenance policy for a system subjected to a shock process. The degradation is modeled as a compound Poisson process with *iid* damage increments. The failures are only detected by inspection, which are also referred to as latent failures in the literature. The inspections were periodic. Detailed applications of the shock process models to the preventive maintenance are given in Nakagawa [4, Chapter 6]. The degradation was modeled as a compound Poisson process and both periodic and sequential inspections were considered. Here both latent and self-announced failures were incorporated in the model. A generalization of damage with annealing (i.e., recovery) was discussed in Chapter 2 of this book. Nakagawa [5, Section 4.3] presented further generalization of the shock process model by considering imperfect maintenance. Another approach was to introduce the probability of failure as a function of the accumulated damage. It means that system's proneness to failure will increase with accumulation of damage. Wang and Pham [6, Section 2] also reviewed shock process models and presented variations of imperfect maintenance and studied dependent damage in standby systems. Aven and Jensen [8, Section 3] presented the Markov modulated process with state-dependent failure probabilities. The computation of discounted cost of maintenance was analyzed by Weide et al. [9]. In the context of compound Poisson process, most researchers analyzed the asymptotic cost rate without discounting. Weide et al. [10] extended this work and presented the formulation for computing discounted cost of a condition-based maintenance program.

In most of the literature surveyed in these monographs, the damage process was primarily taken as a renewal process, and its variants arising from imperfect maintenance were analyzed. These models did not seem to address the modeling of nonlinearities in the damage process that are of concern in this paper. The paper is therefore intended to present a more generalized and conceptually clear analysis of stochastic shock–damage process and apply this for the optimization of maintenance cost.

## 2. Basic approach to maintenance cost evaluation

### 2.1. Maintenance policy

The system damage is modeled as a stochastic cumulative process $Z=\{Z(t); t\geq 0\}$ with increasing sample paths. A failure occurs at time $t$ if the total damage $Z(t)$ exceeds a critical threshold $z_F$. The failure prompts a corrective maintenance (CM) action involving the system renewal through replacement or complete overhaul (as good as new repair). A condition-based preventive maintenance (PM) strategy is adopted in which the system is periodically inspected at an interval $\delta$. The system is renewed at any $j$th inspection time, if $Z(j\delta)$ exceeds a threshold $z_M$, $z_M<z_F$. The time to perform CM and PM are denoted as $r_F$ and $r_M$ (i.e., repair times), respectively. We consider two cases of system failure: (1) a self-announced failure, which immediately triggers CM and (2) a latent failure that is detected only during an inspection. These two cases result in different costs of unavailability.

After each maintenance action, a new cycle starts, independent of the past. Denote the cycles by $(T_1,K_1),(T_2,K_2),\ldots$ where $T_i$ and $K_i$ are the length and the cost of the $i$th renewal cycle. Let $S_i=\sum_{j=1}^{i}T_j$, $i=1,2,\ldots$ be the sequence of partial sums of the cycle lengths $T_i$. At time $S_i$ the maintenance action required at the end of the $i$th cycle is completed. Then the counting process $M=\{M(t): t\geq 0\}$ associated with the sequence $S_i$ is a renewal process and the total cost up to and including time $t$ can be given as

$$K(t)=\sum_{j=1}^{M(t)}K_j. \tag{1}$$

In maintenance, an important question is how to optimize preventive maintenance policy with respect to inspection interval $\delta$ or PM threshold $z_M$. The long-term or asymptotic cost rate $Q=\lim_{t\to\infty}(1/t)K(t)$ (i.e., cost per unit time), has been considered as a basis for this optimization. The basic idea is choose those values of $\delta$ and $z_M$ that would minimize the asymptotic cost rate, for which a well-known result is that [7]

$$Q=\lim_{t\to\infty}\frac{1}{t}K(t)=\frac{E(K)}{E(T)}. \tag{2}$$

The use of asymptotic rate formula leads to considerable simplification, since it requires the evaluation of only the expected cost incurred in a single renewal cycle and its expected duration. In essence, a stochastic repairable process model is replaced by a simple "first failure" or non-repairable process model.

### 2.2. Latent failures

The latent failure means that if a system fails in between an inspection interval, it will remain undetected until the time of the next inspection. In this case, either CM or PM can occur only at the time of any $j$th inspection, denoted as $t_j$, $(j=1,2,\ldots,)$. For a periodic inspection policy, $t_j=j\delta$.

The occurrence of PM or CM depends on the magnitude of the damage in relation to maintenance thresholds. Denoting PM event as $B_j$ and CM as $C_j$, they can be analytically defined as

$$B_j=\{Z(t_{j-1})\leq z_M,\ z_M<Z(t_j)\leq z_F\},$$

$$C_j=\{Z(t_{j-1})\leq z_M,\ Z(t_j)>z_F\},$$

$$A_j=B_j\cup C_j=\{Z(t_{j-1})\leq z_M,\ Z(t_j)>z_M\}. \tag{3}$$

A renewal cycle is a sum of all the time intervals of inspections incurred up to the time at which PM or CM is commenced. In addition, it would include the repair time. Thus, the length, $T$, of a renewal cycle is equal to

$$T=\sum_{j=1}^{\infty}[t_j\mathbf{1}_{A_j}+r_M\mathbf{1}_{B_j}+r_F\mathbf{1}_{C_j}]. \tag{4}$$

The expected (or mean) cycle length can be evaluated as

$$E[T]=\delta\sum_{j=1}^{\infty}ja_j+r_M\sum_{j=1}^{\infty}b_j+r_F\sum_{j=1}^{\infty}c_j, \tag{5}$$

where $a_j=P(A_j)$, $b_j=P(B_j)$, $c_j=P(C_j)$ and $t_j=j\delta$. The infinite sum $\sum_{i=1}^{\infty}b_i$ is the probability that a renewal cycle will end with a PM. Similarly, the sum $\sum_{i=1}^{\infty}c_i$ is the probability that a renewal cycle will end with a CM.