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Merging expert and empirical data for rare event frequency estimation: Pool homogenisation for empirical Bayes models

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ABSTRACT

Empirical Bayes provides one approach to estimating the frequency of rare events as a weighted average of the frequencies of an event and a pool of events. The pool will draw upon, for example, events with similar precursors. The higher the degree of homogeneity of the pool, then the Empirical Bayes estimator will be more accurate. We propose and evaluate a new method using homogenisation factors under the assumption that events are generated from a Homogeneous Poisson Process. The homogenisation factors are scaling constants, which can be elicited through structured expert judgement and used to align the frequencies of different events, hence homogenising the pool. The estimation error relative to the homogeneity of the pool is examined theoretically indicating that reduced error is associated with larger pool homogeneity. The effects of misspecified expert assessments of the homogenisation factors are examined theoretically and through simulation experiments. Our results show that the proposed Empirical Bayes method using homogenisation factors is robust under different degrees of misspecification.

1. Introduction

The estimation of the frequency of rare events is common in Probabilistic Risk Assessment (PRA). Indeed our motivation for this work has been driven by PRA projects in, for example, explosive storage, railway system and power plants [22,12,3,28]. Consider the explosives storage application where we might anticipate a small number of observed events spread over a larger number of incident categories and explosive types. It is unlikely that we will regularly observe data for every combination of incident and explosive type. For example, [18] reported 79 incidents on the UK mainland between 1950 and 1997. Of these 79, only 16 occurred in storage, a rate of one every 3 years (and none of these major incidents).

A variety of inference approaches can be used to estimate the frequency of rare events, including classical statistical methods, direct subjective expert judgement, and both fully Bayesian and Empirical Bayes methods. Each has its strengths and limitations. For example, the classical approach is to estimate the rate as a ratio of the number of observed events to the exposure time. For situations where no events are observed, a variety of ways are proposed to adjusting the otherwise overoptimistic point estimate of zero. These include: substituting the zero estimate of the rate by the value of the reciprocal of the exposure time [1]; using the upper bounds of Chi-Squared [7] or Normal [29] confidence intervals for the rate; or minimising the maximum expected squared error [23]. See Quigley and Revie [23], Bailey [1] and Williams and Thorne [29] for further review and comparison of alternative classical approaches for this problem. Generally we believe that classical statistical estimators for rare event frequencies may not be very useful because the uncertainty associated with the estimates obtained will be relatively large.

Bayesian methods are widely used in PRA [15] and allow the empirical data to be balanced against the subjective beliefs of experts. Practical examples of the use of full Bayes methods are given by [19,4] and [17]. A Bayesian paired comparison method for estimating rare event probabilities is considered by [26,19] discussing common objections to the Bayesian approach. In our context, fully Bayesian methods require the specification of a subjective prior distribution, which may not only be difficult to achieve but which will largely determine the outcome of the estimation process, because of relatively small amount of data is available with which to update the prior. Hence the estimates will be highly influenced by the initial subjective engineering judgement. This limitation is also relevant to the direct use of expert judgement to obtain a point estimate [2].

In [22] we have previously explored the use of Empirical Bayes, a hybrid of classical and Bayesian methods in which the prior is found using empirical methods. The Empirical Bayes method

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Nomenclature

m	number of processes in the pool
Λ_i	rate of occurrence of process $i$modelled as a random variable described by the prior distribution
λ_i	realisation of the rate of occurrence of process i
α	shape parameter of prior distribution when not explicitly modelling heterogeneity
β	scale parameter of prior when not explicitly modelling heterogeneity
a	shape parameter of prior distribution when explicitly modelling heterogeneity
b	scale parameter of prior distribution when explicitly modelling heterogeneity
$\hat{a}$	estimator of a
$\widehat{b}$	estimator of b
n_i	number of events in process i
$\underline{n}$	vector of number of events each process
t	exposure time for each process (i.e. common for all processes)
h_i	homogenisation factor for process i
$\overline{h}$	mean of homogenisation factors
$\tilde{h}_i$	subjective expert assessment of homogenisation factor for process i
$\overline{h^2}$	mean of squared homogenisation factor
r	ratio of $\overline{h}/\overline{h^2}$
U	adjusted rate of occurrence of events for all processes
W	information about the second moment within the pool of all processes
S_i	multiplicative random error in subjective assessment of homogenisation factor h_i
θ	mean of multiplicative error in subjective assessment of homogenisation factor h_i
σ^2	variance of multiplicative error in subjective assessment of homogenisation factor h_i
μ_i	expected rate of occurrence of process i, given parameter values of prior distribution are known
$\mu_{h,i}$	expected rate of occurrence of process i, given homogenisation factors are used
$\hat{\mu}_i$	estimator of expected rate of occurrence of process $i$when parameter values of prior distribution are known and so homogenisation factor not used
$\hat{\mu}_{h,i}$	estimator of expected rate of occurrence of process type $i$when homogenisation factors are used
k_i	logarithm (base 10) of the homogenisation factor for process i
$\tilde{k}_i$	logarithm (base 10) of the subjective expert assessment of homogenisation factor for process i
e_i	arithmetic error in estimating the rate of occurrence of process i

allows the pooling of observed data across multiple events to estimate an overall rate. Individual occurrence rates are then calculated as deviations from this overall rate. See [6] for a general overview of Empirical Bayes. Unlike Bayesian methods, Empirical Bayes is not a fully subjective approach because it uses the pooled data to estimate the prior distribution parameters. It can be argued that Empirical Bayes possesses some of the benefits of Bayesian methods while avoiding the need for subjective specification of a prior distribution. By relying on empirical data to form the prior, we can retain some of the benefits of using historical data when estimating rare event frequencies as discussed by [9].

Empirical Bayes methods have been applied in the fields of reliability [24] and risk analysis [16] and are regularly used to analyse accident occurrence patterns in road safety applications [21,8]. Empirical Bayes models have been shown to perform well against full Bayes models [5], and even favourably when there are few observed data [27,25]. However, the Empirical Bayes method also has limitations. For example, it has not been established how to formally incorporate quantitative expert judgement and the accuracy of the estimates obtained depend on the degree of homogeneity of the pool of events used to form the prior; for example, the more homogeneous the pool of events, then we might expect the estimate obtained to be more accurate.

In this paper we discuss a novel integration of quantitative expert judgement into Empirical Bayes to homogenise the pool of events used to form the prior and so meet the goal of obtaining more accurate estimates of the rate of occurrence of rare events. Our method does not require an expert to assess the absolute values of frequencies, but merely to assess relative rates. Hence the method could be operationalised by using methods such as pairwise comparison [20,10] that naturally give ratios rather than absolute values.

Our proposed approach aims to use expert information to rescale data by effectively choosing a natural time scale for each event type in order to improve the behaviour of the Empirical Bayes estimator. Expert judgement is introduced to assess the so-called homogenisation factors, which are scaling constants to bring the frequencies of different event types to approximately the same value. By using such homogenisation factors the estimates produced by pooling different event types using Empirical Bayes should become more accurate, in the sense that a more homogenous pool will attach more weight to the pooled average rather than the individual event experience. A reduction in error will be a consequence of greater reliance on a representative estimate derived from a larger sample size.

In this paper we develop the proposed method under the assumption that the events are generated from a Homogeneous Poisson Process (HPP), which is a not unreasonable model for the case where the rate of events can be treated as constant. Although the homogenisation factors are unknown constants, we consider the epistemic uncertainty in assessing them as random variables. We examine the estimation error relative to the homogeneity of the pool to provide an assessment of the accuracy of our proposed estimators. Given that we advocate that the homogenisation factors are obtained using expert judgement, we evaluate the impact of poor subjective assessments on the robustness of our estimates. Therefore, in summary, this paper contributes a new approach to Empirical Bayes estimation for rare event frequencies and examines the properties of the proposed method both theoretically and through simulation experiments.

The modelling framework adopted for this problem and the development of the new Empirical Bayes estimator are described in detail in Section 2. Section 3 reports a theoretical investigation of the impact of misspecification of homogenisation factors on inference when the Method of Moments is used for parameter estimation. Section 4 presents the design and results of a simulation study to explore the key aspects of the model set-up under Maximum Likelihood and controlled pool sizes. Section 5 presents concluding remarks and discusses further work.

2. Model formulation and inference

We consider a set of processes, each generating data according to a Homogeneous Poisson Process (HPP) but not necessarily at