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Parameter-robust linear quadratic Gaussian technique for multi-agent slung load transportation

Hae-In Lee [a], Dong-Wan Yoo [b], Byung-Yoon Lee [c], Gun-Hee Moon [c], Dong-Yeon Lee [c], Min-Jea Tahk [c,*]

[a] *Cranfield University, College Road, Cranfield, Bedfordshire, MK43 0AL, UK*
[b] *The Agency of Defence Development, Yuseong, Daejeon, Republic of Korea*
[c] *KAIST, 291 Daehak-ro, Daejeon, 305-701, Republic of Korea*

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ABSTRACT

This paper copes with parameter-robust controller design for transportation system by multiple unmanned aerial vehicles. The transportation is designed in the form of string connection. Minimal state-space realization of slung-load dynamics is obtained by Newtonian approach with spherical coordinates. Linear quadratic Gaussian / loop transfer recovery (LQG/LTR) is implemented to control the position and attitude of all the vehicles and payloads. The controller's robustness against variation of payload mass is improved using parameter-robust linear quadratic Gaussian (PRLQG) method. Numerical simulations are conducted with several transportation cases. The result verifies that LQG/LTR shows fast performance while PRLQG has its strong point in robustness against system variation.

1. Introduction

Recently, unmanned aerial vehicles (UAVs) are getting attention for both military and civilian uses. In the future, transportation using UAVs is also expected to be common, but a small light UAV generally does not have enough power to lift a heavy load. Rather than employing a larger UAV, cooperation of multiple UAVs can be an efficient approach for transporting various types of payload. Interconnection of multiple UAVs results in complicated equations of motion, as each UAV heavily affects the motion of the others. Importance and possibility of employing multiple UAVs in transportation has been mentioned in other previous studies [1]. Although there are many possible ways of cooperation, such as rigid gripping with clamps [2] and bar joint, string connection is chosen in this study to maintain the degree of freedom of each UAV as shown in Fig. 1.

In the previous studies, a single aircraft lifting one payload with a long string has been considered [3,4]. As these cases assume a sufficiently long pendulum, coupling effects or aerodynamic disturbances on payload are negligible, and thus whole system does not have to be included in the model. To consider

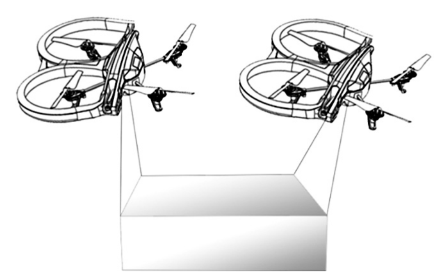

Fig. 1. Transportation system with string connections.

coupling effects into model, Maza et al. solved this problem using Kane method [5–7]. On the other hand, Bisgaard et al. [8,9] employed Udwadia–Kalaba Equation (UKE), which is more efficient in expressing constrained dynamics. Existing modeling techniques, both Kane and UKE method, give the precise model of slung load transportation system, while state-space representations are not minimal. As one string connection reduces one degree of freedom, the minimum number of states is reduced by the number of strings. The existence of superficial states leads to a system model absent of controllability. Since the number of system variables is large and the model is complicated, model reduction is not easy. Our previous work [10] using UKE method, therefore, was not able to apply LQ-based controllers. To circumvent the controllability problem, it is suggested to utilize the combination of

* Corresponding author.
E-mail addresses: haein.lee@cranfield.ac.uk (H.-I. Lee), dwyoo@fdcl.kaist.ac.kr (D.-W. Yoo), bylee@fdcl.kaist.ac.kr (B.-Y. Lee), ghmoon@fdcl.kaist.ac.kr (G.-H. Moon), dylee@fdcl.kaist.ac.kr (D.-Y. Lee), mjtahk317@gmail.com (M.-J. Tahk).

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spherical coordinates and Cartesian coordinates [11]. The equation of motion is derived by Newtonian approach as it is easier to generalize the equation of motion compared to the Lagrangian approach. Unlike previous methods, tensile forces are computed by matrix inversion with inclusion of internal force into the state vector.

Stability analysis and control of the modeled system is another major issue in this paper. Two approaches are possible for control design: control of each UAV with respect to external disturbance including the effect of tension and control design considering the whole system. Previously, studies in [3,4] utilize the former and Michael et al. [12,13] perform only stability analysis. In order to conduct aggressive control in response to pendulum motion, whole system states are required to be controlled at the same time. This paper implements classical optimal control technique, linear quadratic Gaussian (LQG) technique [14]. LQG is useful to find gains for complicated transportation systems, while PID control, the most commonly used method, is hard to be implemented for large number of states. The tuning of PID gains is generally performed by trial-and-error and coupling between the longitudinal and lateral dynamics makes this tuning hard.

For practical use, it would be better to transport the payloads with various weights without changing the controller. Also, continuous loss of weight during transportation is common in agricultural uses. To improve the robustness of the LQG controller, loop transfer recovery (LTR) [15,16] or parameter-robust linear quadratic Gaussian (PRLQG) [17,18] can be employed. PRLQG is expected to provide better robustness then LQG/LTR. In addition to our previous work [11], frequency-domain analysis on stability proves the improvements in robustness.

This paper is composed as follows. First, the mathematical modeling procedure of multi-UAV slung load transportation system using Newtonian approach is presented. Second, control design theory of LQG/LTR and PRLQG method is briefly reviewed, and the transportation system model is reformulated into a moderate form for controller design process. Third, numerical simulation using MATLAB is conducted to analyze the performance of LQG/LTR and PRLQG controller. Finally, conclusion is drawn from numerical results and future work is suggested.

2. Slung load transportation system modeling

The following sub-sections suggest modeling procedure for transportation system with Newtonian approach, assuming no aerodynamic force or fluctuation in strings. Only gravitational force and lift force of UAVs are assumed to be significant in the model. The equation of motion is generalized with unspecified number and shape of UAVs. General equation of motion is then reduced to two cases: one point mass transportation system with one UAV, and one box payload transportation with four UAVs.

2.1. Derivation of general equation of motion

System variables of transportation system with unspecified number of UAVs and type of payloads are shown in Fig. 2. System variables chosen for modeling are position and attitude of the load ($\mathbf{x}_L$, $\boldsymbol{\theta}_L$), spherical coordinate angle of the strings ($\boldsymbol{\theta}_i$), and attitude of each UAV ($\boldsymbol{\theta}_{V,i}$), where attitude information is required for computing direction cosine matrices ($\mathbb{C}^E_{V,i}$, $\mathbb{C}^E_L$) and spherical coordinate (C) is used to describe the motion of strings for constrained length. The spherical coordinate is determined so that zero angles yield hovering condition as follows:

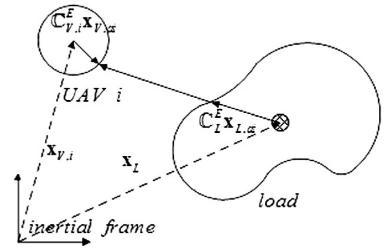

Fig. 2. Transportation system nomenclature.

$$C(\boldsymbol{\theta}) = l\begin{bmatrix} \sin\phi \\ -\sin\theta\cos\phi \\ -\cos\theta\cos\phi \end{bmatrix} \tag{1}$$

Observing the geometric relationship in Fig. 2, position states of the UAVs ($\mathbf{x}_{V,i}$) are determined as follows:

$$\mathbf{x}_{V,i} = \mathbf{x}_L + \mathbb{C}^E_L\mathbf{x}_{L,ai} + C(\boldsymbol{\theta}_i) - \mathbb{C}^E_{V,i}\mathbf{x}_{V,ai} \tag{2}$$

where $\mathbf{x}_{v,ai}$ and $\mathbf{x}_{L,ai}$ stands for the vectors from the center of mass of vehicle or load respectively to the attachment point of i-th string.

Applying Newton's 2nd law of motion and Euler equation, the following equation is the basic idea of modeling:

$$\begin{cases} \mathbf{M}_{V,i}\ddot{\mathbf{x}}_{V,i} & = \mathbf{F}_{V,i} - \sum T_i C(\boldsymbol{\theta}_i)/l_i \\ \mathbf{M}_L\ddot{\mathbf{x}}_L & = \mathbf{F}_L + \sum T_i C(\boldsymbol{\theta}_i)/l_i \\ \mathbf{I}_V\dot{\boldsymbol{\omega}}_{V,i} & = \boldsymbol{\tau}_{V,i} - \sum T_i(\mathbf{x}_{V,ai} \times \mathbb{C}^{V,i}_E C(\boldsymbol{\theta}_i))/l_i \\ \mathbf{I}_L\dot{\boldsymbol{\omega}}_L & = \sum T_i(\mathbf{x}_{L,ai} \times \mathbb{C}^L_E C(\boldsymbol{\theta}_i))/l_i \end{cases} \tag{3}$$

where $\mathbf{M}_L$ or $\mathbf{M}_{V,i}$ is a mass matrix with diagonal entries of mass m_L or $m_{V,i}$, $\mathbf{I}$ is an inertial matrix, T is a tensile force, l is the length of a string, and $\boldsymbol{\omega}$ is the angular velocity in the body frame. The forces $\mathbf{F}_{V,i}$ and $\mathbf{F}_L$ include the gravitational force as

$$\mathbf{F}_{V,i} = \mathbb{C}^E_{V,i}\mathbf{F}_{M,i} + \mathbf{M}_{V,i}\mathbf{g}, \quad \mathbf{F}_L = \mathbf{M}_L\mathbf{g}, \quad \mathbf{g} = [0, 0, g]^T \tag{4}$$

The inputs of the system are given as external forces ($\mathbf{F}_{M,i}$) and moments ($\boldsymbol{\tau}_{V,i}$) regardless of UAV dynamics. This modeling is for general type and dynamics of UAV, and thus individual dynamics must be augmented separately.

Substitution of equation (2) into equation (3) yields relationship among system variables and tensile forces. Although values of tensile forces are not measured, second-derivative terms and tensile forces have linear relationship, and thus derivative terms can be computed as an inverse matrix form as follows:

$$\begin{bmatrix} \ddot{\mathbf{x}}_L^T & \ddot{\boldsymbol{\theta}}_i^T & \dot{\boldsymbol{\omega}}_L^T & \dot{\boldsymbol{\omega}}_{V,i}^T & T_i/l_i \end{bmatrix}^T = \begin{bmatrix} \mathbf{M}_{V,i} & \mathbf{M}_{V,i}C'(\boldsymbol{\theta}_i) & \mathbf{M}_{V,i}\mathbb{C}'^E_L\mathbf{x}_{L,ai} & -\mathbf{M}_{V,i}\mathbb{C}'^E_{V,i}\mathbf{x}_{V,ai} & C(\boldsymbol{\theta}_i) \\ \mathbf{M}_L & \mathbf{0}_{3\times2} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & -C(\boldsymbol{\theta}_i) \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times2} & \mathbf{0}_{3\times3} & \mathbf{I}_{V,i} & \mathbf{x}_{V,ai} \times \mathbb{C}^{V,i}_E C(\boldsymbol{\theta}_i) \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times2} & \mathbf{I}_L & \mathbf{0}_{3\times3} & -\mathbf{x}_{L,ai} \times \mathbb{C}^L_E C(\boldsymbol{\theta}_i) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{F}_{V,i} - \sum_i \mathbf{M}_{V,i}(G(\boldsymbol{\theta}_i, \dot{\boldsymbol{\theta}}_i) + \mathbb{G}^E_L\mathbf{x}_{L,ai} - \mathbb{G}^E_{V,i}\mathbf{x}_{V,ai}) \\ \mathbf{F}_L \\ \boldsymbol{\tau}_{V,i} \\ \mathbf{0}_{3\times1} \end{bmatrix} \tag{5}$$

The prime mark notes for differentiation not along the time but along the spherical coordinate states of strings. To be specific, function C' and matrix $\mathbb{C}'$ is computed as

$$C' \triangleq \begin{bmatrix} \frac{dC}{d\theta} & \frac{dC}{d\phi} \end{bmatrix}, \quad \mathbb{C}'\mathbf{x}_{ai} \triangleq \begin{bmatrix} \frac{d\mathbb{C}}{d\phi}\mathbf{x}_{ai} & \frac{d\mathbb{C}}{d\theta}\mathbf{x}_{ai} & \frac{d\mathbb{C}}{d\psi}\mathbf{x}_{ai} \end{bmatrix} \tag{6}$$

where the differentiated result is similar to the gradient in that the required form is an augmented matrix.