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An efficient multiple point selection study for mesh deformation using radial basis functions

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A B S T R A C T

The mesh deformation using radial basis functions (RBFs) is widely used in aerodynamic applications in the context of design & optimization and aeroelasticity. One reason is the good performance for test cases requiring only a few number of support points. But in test cases with complex deflection fields, requiring a large number of support points, the performance of the method degenerates. To improve an existing RBFs-based mesh-deformation method, an efficient multiple point selection method has been developed based on the conventional Greedy method. The interpolation error function of the RBFs-based model is analyzed, and multiple points with maximum local error are selected simultaneously into the support set, with which an interpolation model will be finally built to compute the displacements of the volume mesh nodes. To avoid too many redundant points for building the interpolation model, a threshold factor is conceived to limit the number of points selected. The computational cost by the Greedy method, the multi-level data reduction method and the developed method, are theoretically compared. Typical deformation problems are chosen as test cases for demonstrating current implementations, like a ninety-degree rotation of a NACA 0012 airfoil, the adjustment of the position of the slat and the flap of a high-lift airfoil, the bending of the DLR F6 configuration, and the shape modification of a hypersonic Rocket-Based Combined Cycle space vehicle. Further, the effect of the threshold factor is studied. The results show that the developed mesh-deformation method has a better efficiency than the Greedy method and the multi-level data reduction method, especially for test cases with complex deflection fields.

1. Introduction

In the context of design and optimization of modern flight vehicles [1–3], new or adjusted meshes have to be provided, if the shape changes. They can be obtained by two ways: mesh regeneration and mesh deformation. The mesh-regeneration method is more flexible, but is computationally expensive for complex configurations with large number of mesh nodes. In comparison, the mesh-deformation method only transmits the displacement of the configuration surface into the volume mesh nodes, without changing the scale and the topology of the baseline mesh. Thus, the mesh-deformation method is more efficient and also suitable for aeroelastic simulations and automated optimization processes.

Currently, many mesh-deformation methods are in use. Gaitonde et al. developed a polynomial interpolation method [4,5], which is well suited for structured meshes with simple topologies. The spring interpolation, the elastic solid method and the Delaunay graph mapping are suitable for unstructured as well as for structured meshes. The spring interpolation was first proposed by Batina [6,7], in which the mesh is regarded as a network of springs. Hence the new coordinates of the mesh nodes can be computed by solving the force-equilibrium equations of the network. This method is robust but inefficient for complex configurations. The elastic solid method [8,9] treats the mesh like an elastic solid and the displacement of the boundary as loads on the solid, thus new mesh can be obtained by solving the equilibrium equations of the elastic solid. This method can handle large deformation, but has low efficiency with regard to large number of mesh nodes. The Delaunay graph mapping [10,11] transfers the displacement of the surface into the background mesh, then updates the coordinates of the mesh nodes based on the background mesh. This method is efficient, simple, and suitable for meshes with different topologies. But it is limited to configurations with convex boundaries and good background meshes, otherwise the deformed mesh will be in bad quality.

In recent years, the mesh-deformation method using RBFs becomes more popular [12–17]. Boer was one of the first to have

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applied the RBFs in mesh deformation, by building the RBFs-based model using the surface mesh nodes and computing new mesh coordinates through the model [18]. The mesh-deformation method based on the RBFs has following merits: a) there is no need of topology information of the mesh, and the data structure is simple; b) the method is suitable for any type of mesh such as structured, unstructured or structured/unstructured hybrid mesh; c) it can handle complex deflection fields. Along with increasing the number of the surface mesh nodes, the efficiency of the RBFs-based mesh-deformation method decreases fast. One way to solve this problem is to select just a set of surface mesh nodes as support points building the RBFs-based model. Rendall et al. [19–23] has used the Greedy method to select the surface mesh nodes with maximum error in each iteration step to build the RBFs-based model, trying to reduce the number of support points on the premise of a good interpolation accuracy. Wang et al. [24,25] introduced a subspace into the Greedy method, and developed the multi-level data reduction (MLDR) method, which can select support points more efficiently. Besides, Xie et al. [26] improved the deformation efficiency further by employing the subspace into updating the volume mesh coordinates. Liu et al. developed a two-step mesh-deformation strategy based on the RBFs [27]. Gillebaart et al. have proposed an adaptive RBFs-based mesh-deformation method by keeping track of the boundary error throughout the simulation and the re-selection [28].

In the present work, we propose an efficient multiple point-selection method based on the Greedy method for mesh deformation using RBFs. The idea is to build a RBFs-based model at first, then identify multiple peaks of the error function for the interpolation model, which are used to expand the set of support points for constructing RBFs-based models. After some iterations, the RBFs-based model satisfies the specified error criterion. Then the model can be finally used to compute the displacements of the volume mesh nodes. The theory of the mesh deformation based on the RBFs is briefly explained in Section 2. The developed mesh-deformation method is detailed in Section 3. In Section 4, the advantages of the developed method are discussed and demonstrated using four applications: ninety-degree rotation of the NACA 0012 airfoil with an unstructured mesh; adjusting the setting of the slat and the flap of a three-element high-lift airfoil with a hybrid mesh; bending the DLR F6 configuration with an unstructured/structured hybrid mesh; changing the shape of a configuration by modifying several design variables of a hypersonic Rocket-Based Combined Cycle (RBCC) space vehicle with a structured mesh. The effect of the threshold factor on deforming the shapes by the developed method is studied. Finally, conclusions are drawn in Section 5.

2. Mesh deformation based on radial basis functions

The basic form of the RBFs-based model using for mesh deformation can be written as

$$\boldsymbol{F}(\boldsymbol{r}) = \sum_{i=1}^{N_b} \omega_i \varphi\left(\|\boldsymbol{r} - \boldsymbol{r}_i\|\right) \tag{1}$$

where, $\boldsymbol{F}(\boldsymbol{r})$ is the displacement vector at the position $\boldsymbol{r}$; N_b is the number of the selected support points; i is the index of the support point; $\boldsymbol{r}_i$ is the position vector of the ith support point; ω_i is the weight coefficient according to the ith support point; $\|\boldsymbol{r} - \boldsymbol{r}_i\|$ is the norm between the mesh node and the support point respectively located at $\boldsymbol{r}$ and $\boldsymbol{r}_i$; $\varphi(\|\boldsymbol{r} - \boldsymbol{r}_i\|)$ is the general form of a certain kind of radial basis function adopted. In comparison with the global RBFs, the compactly supported positive definite basis function is suitable for problems with large number of mesh nodes [29]. Thus, the Wendland's series are chosen as the basis functions [30].

In the RBFs-based mesh deformation, the surface mesh nodes are used as the support points for building the interpolation model. The model can be described by

$$\Delta \boldsymbol{S}_b = \boldsymbol{\Phi}_b \boldsymbol{W} \tag{2}$$

where, $\Delta \boldsymbol{S}_b$ is the displacement matrix of the mesh nodes on the surface, expressed as $\Delta \boldsymbol{S}_b = \{\Delta \boldsymbol{s}_1, \Delta \boldsymbol{s}_2, \ldots, \Delta \boldsymbol{s}_{N_b}\}^T$, of which $\Delta \boldsymbol{s}_i = \{\Delta x_i, \Delta y_i, \Delta z_i\}^T$ is the displacement vector of the ith surface mesh node. $\boldsymbol{W}$ is the weight-coefficient matrix $\{\boldsymbol{\omega}^x, \boldsymbol{\omega}^y, \boldsymbol{\omega}^z\}$ to be solved, of which $\boldsymbol{\omega}^x = \{\omega_1^x, \omega_2^x, \ldots, \omega_{N_b}^x\}^T$, $\boldsymbol{\omega}^y = \{\omega_1^y, \omega_2^y, \ldots, \omega_{N_b}^y\}^T$, and $\boldsymbol{\omega}^z = \{\omega_1^z, \omega_2^z, \ldots, \omega_{N_b}^z\}^T$. $\boldsymbol{\Phi}_b$ is the basis matrix containing basis functions, defined as

$$\boldsymbol{\Phi}_b = \begin{Bmatrix} \varphi_{1,1} & \cdots & \varphi_{1,i} & \cdots & \varphi_{1,N_b} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_{j,1} & \cdots & \varphi_{j,i} & \cdots & \varphi_{j,N_b} \\ \vdots & \vdots & \vdots & \vdots & \vdots \cdots \\ \varphi_{N_b,1} & \cdots & \varphi_{N_b,i} & \cdots & \varphi_{N_b,N_b} \end{Bmatrix} \tag{3}$$

where, $\varphi_{j,i} = \varphi(\|\boldsymbol{r}_j - \boldsymbol{r}_i\|)$ is the basis function between the surface mesh nodes j and i. Hence the weight coefficient matrix can be obtained by solving Eq. (2). Further, the displacements of the volume mesh nodes can be calculated using the interpolation model

$$\Delta \boldsymbol{S}_v = \boldsymbol{\Phi}_v \boldsymbol{W} \tag{4}$$

where, $\boldsymbol{\Phi}_v$ is built by all the volume mesh nodes and the surface mesh nodes, expressed as $\boldsymbol{\Phi}_v = \{\boldsymbol{\varphi}_{j,1}, \boldsymbol{\varphi}_{j,2}, \ldots, \boldsymbol{\varphi}_{j,N_b}\}$ and $j = 1, 2, \ldots, N_v$, where N_v is the number of the volume mesh nodes. Thus, the new coordinates of the volume mesh nodes can be updated by

$$\boldsymbol{S}_{new} = \boldsymbol{S}_{old} + \Delta \boldsymbol{S}_v \tag{5}$$

3. Efficient multiple point-selection method

The computational cost of the RBFs-based mesh-deformation method depends on the number of the surface mesh nodes N_b and the number of the volume mesh nodes N_v, according to Eq. (2) and Eq. (4). For standard problems, Eq. (2) has to be solved only once, and the solution can then be reused for a modified deflection field. However, if the configuration is complicated and the deflection field is complex, a large number of surface and volume mesh nodes are involved in solving Eq. (2). Hence the conventional RBFs-based mesh-deformation method is infeasible because of the huge computational cost. Thus, the idea to solve the above problem is to select just a set of surface mesh nodes as support points, which will be used to build a reduced-order RBFs-based model with a promise of sufficient model accuracy. Therefore, the deformation process can be more efficient, and can guarantee a good quality of the deformed surface mesh. The Greedy method by Rendall [21] generates an initial support set of samples to fit the RBFs-based model, then adds one surface mesh node with maximum interpolation error into the support set gradually in the following iterations until the RBFs-based model is sufficient accurate. Thus, the final set of the support points can be used to build an interpolation model to compute the displacements of the volume mesh nodes. For problems with tens of thousands surface mesh nodes, the Greedy method converges quite slowly.

We developed an efficient multiple point selection method based on the Greedy method. The multiple nodes with maximum local interpolation error are selected to expand the support set simultaneously in each iteration step, in order to improve the efficiency of identifying a support set that can be used to build an

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