Aerospace Science and Technology 54 (2016) 108–113

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Short communication

Attitude control using higher order sliding mode

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ARTICLE INFO

Article history:
Received 14 January 2015
Received in revised form 16 March 2016
Accepted 11 April 2016
Available online 18 April 2016

Keywords:
Attitude control
Higher order sliding mode
Integral sliding mode
Finite-time convergence

ABSTRACT

This paper proposes a chattering-free global finite-time convergent robust control method for the attitude control of a rigid spacecraft. The control method is formulated using higher order sliding mode control in integral sliding mode. The sliding mode order is higher than the system relative degree and therefore, chattering elimination is ensured. In the control design process, unmeasured sliding vector derivative is estimated using a robust exact differentiator. To verify the effectiveness of the proposed control method, simulations are performed in the presence of external disturbances and inertia uncertainty. The proposed controller successfully negates the effect of external disturbances and inertia uncertainty and attitude states attain the desired accuracy with good steady state precision and high convergence speed.

1. Introduction

In a space mission, attitude control design is a high-priority issue and mostly a mission success is greatly linked with attitude control performance. To summarize, pointing accuracy, convergence time, robustness against uncertainty and disturbances, and control energy requirement are the major basis to evaluate the attitude control performance.

Almost all nonlinear control design techniques have been investigated to tackle the control related issues present in an attitude control design. However, sliding mode control (SMC), because of inherent robustness property [1,2], has invited more attention compared to others. The significant contribution of SMC in attitude control can be seen in [3–6]. However, the standard SMC's two significant limitations are asymptotic convergence in sliding mode and chattering in control. In majority of contributions, to limit chattering level, discontinuous $sign(.)$ has been approximated with $sat(.)$ or $tanh(.)$. However, doing this has certainly weaken the controller robustness. The second limitation, asymptotic convergence in sliding mode, has been handled through terminal sliding mode (TSM) [7–9] and its variants [12–14]. The major contributions in attitude control by TSM and its variants can be found in [15–17]. Certainly, TSM control application has given a new dimension to attitude control subject and it can be verified that TSM can enhance convergence speed and steady state precision. However, TSM in standard sliding mode cannot guarantee a chattering-free global robust control.

Higher order sliding mode control (HOSMC), another advancement in sliding mode, is an instrumental technique to eliminate chattering and ensure zero in finite time [10,11]. In the contrary to standard SMC approach, in HOSMC the discontinuous control is applied into higher order derivative of sliding surface and this way can control chattering. Taking HOSMC, different nonlinear problems have been tackled successfully. HOSMC application in attitude control is not common, and taking HOSMC, two contributions have been found in [18,19]. Nonetheless, higher order sliding mode is not established [20].

Recently, integral sliding mode [21] and higher order sliding mode have been applied together and an integral higher sliding mode control (IHOSMC) method is presented. An IHOSMC, formulated using a proper sliding variable, can fulfill the requirement of a global robust chattering-free control. IHOSMC application is investigated for many nonlinear systems and reported in [22–25]. IHOSMC consideration for attitude control can be seen in [26]. However, chattering is not eliminated.

In this paper, our endeavor is to extend the work reported in [26]. The proposed method presents an attitude controller in integral higher order sliding mode with a robust exact differentiator. The differentiator helps to derive higher order derivative of sliding surface. In control formulation, control input is introduced as a new auxiliary variable and its first time derivative is run by discontinuous input. As a result, a finite-time convergent, globally robust, chattering-free attitude control can be possible. The rest of

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http://dx.doi.org/10.1016/j.ast.2016.04.012
1270-9638/

the paper is organized as follows. In Section 2, spacecraft attitude dynamics and kinematics are explained. The problem formulation and the proposed control design are discussed in Section 3. The closed-loop stability is discussed in Section 4. In Section 5, simulation results have been discussed. The paper ends with concluding remarks in Section 6.

2. System description

Using the unit quaternion, a rigid body spacecraft attitude kinematics can be defined as [29]

$$\dot{q}_v = \frac{1}{2}(q_4 I + q_v^\times)\omega$$
$$\dot{q}_4 = -\frac{1}{2}q_v^T\omega \tag{1}$$

where $q_v = [q_1\ q_2\ q_3]^T \in \Re^3$ and $q_4 \in \Re$ are the vector and scalar components of the unit quaternion $q = [q_v^T\ q_4]^T$, respectively, $\omega \in \Re^3$ is the body angular velocity, and I is the identity matrix of size 3×3. The unit quaternion components are constrained with

$$q_v^T q_v + q_4^2 = 1. \tag{2}$$

For any vector $m = [m_1\ m_2\ m_3]^T \in \Re^3$, notation $m^\times$ is defined by

$$m^\times = \begin{bmatrix} 0 & -m_3 & m_2 \\ m_3 & 0 & -m_1 \\ -m_2 & m_1 & 0 \end{bmatrix}$$

Rigid spacecraft attitude dynamics is defined by

$$\dot{\omega} = J^{-1}(-\omega^\times J\omega + u(t) + d(t)) \tag{3}$$

where $J \in \Re^{3\times3}$ represents the mass inertia matrix with nominal component $J_0 \in \Re^{3\times3}$ and uncertain term $\delta J \in \Re^{3\times3}$, $u(t) \in \Re^3$ is the control input, and $d(t) \in \Re^3$ symbolizes the all external disturbances affecting to the body.

Further, error quaternion $q_e = [q_{ev}^T\ q_{e4}]^T \in \Re^3 \times \Re$ and angular velocity error $\omega_e \in \Re^3$ are measured from body fixed reference frame to the desired reference frame. The defining equations are as follows

$$q_{ev} = q_{d4}q_v - q_{dv}^\times q_v - q_4 q_{dv}$$
$$q_{e4} = q_{dv}^T q_v + q_4 q_{dv}$$
$$\omega_e = \omega - C\omega_d, \tag{4}$$

where $q_{ev} = [q_{e1}\ q_{e2}\ q_{e3}]^T$ and q_{e4} are the vector and scalar components of the error quaternion, respectively, $q_{dv} = [q_{d1}\ q_{d2}\ q_{d3}]^T \in \Re^3$, $q_{d4} \in \Re$, and $\omega_d = [\omega_{d1}\ \omega_{d2}\ \omega_{d3}]^T \in \Re^3$ are the desired attitude frame vector quaternion, scalar quaternion, and angular velocity, respectively. Both q_e and $q_d = [q_{d1}\ q_{d2}\ q_{d3}\ q_{d4}]^T$ satisfy (2). $C = (q_{e4}^2 - 2q_{ev}^T q_{ev})I + 2q_{ev}q_{ev}^T - 2q_{e4}q_{ev}^\times \in \Re^{3\times3}$ with $||C|| = 1$ and $\dot{C} = -\omega_e^\times C$ represents the rotation matrix between body fixed reference frame and desired reference frame.

Using the relation (4), attitude tracking kinematics and the dynamics equations are as follows

$$\dot{q}_{ev} = \frac{1}{2}(q_{e4}I + q_{ev}^\times)\omega_e$$
$$\dot{q}_{e4} = -\frac{1}{2}q_{ev}^T\omega_e \tag{5}$$

$$\dot{\omega}_e = J^{-1}\Big(-(\omega_e + C\omega_d)^\times J(\omega_e + C\omega_d) + J(\omega_e^\times C\omega_d - C\dot{\omega}_d) + u(t) + d(t)\Big). \tag{6}$$

Assumption 1. Throughout the space mission, the attitude describing states q and ω are measurable, and available for the feedback in control design.

Assumption 2. The external disturbance $d(t)$ and its first time derivative are bounded, and their bound limits are known in advance.

Assumption 3. The mass inertia nominal component J and uncertain component δJ are bounded.

Control objective

The control objective is to propose a robust control method to ensure

$$\begin{cases} \lim_{t\to t_f} q_e = [0\ 0\ 0\ 1]^T \\ \lim_{t\to t_f} (\omega_e) = [0\ 0\ 0]^T \end{cases} \tag{7}$$

in finite time t_f.

3. Problem formulation and control design

3.1. Problem formulation

To design a higher order sliding mode based attitude control, the obvious choice is to select sliding surface

$$\sigma = q_{ev}. \tag{8}$$

It is easy to verify that the second derivative of sliding surface yields

$$\ddot{\sigma} = -\frac{1}{4}q_{ev}^T\omega_e\omega_e + \frac{1}{4}T(q_e)\omega_e^\times\omega_e + \frac{1}{2}T(q_e)\dot{\omega}_e, \tag{9}$$

where $T(q_e) = (q_{e4}I + q_{ev}^\times)$. Using (6), we have

$$\ddot{\sigma} = -\frac{1}{4}q_{ev}^T\omega_e\omega_e + \frac{1}{4}T(q_e)\omega_e^\times\omega_e + \frac{1}{2}T(q_e)J^{-1}(-(\omega_e + C\omega_d)^\times J(\omega_e + C\omega_d) + J(\omega_e^\times C\omega_d - C\dot{\omega}_d) + u(t) + d(t)) \tag{10}$$

Now applying the matrix inversion lemma [20], $J^{-1} = (J_0 + \delta J)^{-1} = J_0^{-1} + \widehat{\delta J}$, where $\widehat{\delta J} = -J_0^{-1}\delta J(I_{3\times3} + J_0^{-1}\delta J)^{-1}J_0^{-1}$, (10) gives

$$\ddot{\sigma} = A_{nom}(q_e, \omega_e) + B_{nom}(q_e, \omega_e)u(t) + \bar{d}(t) \tag{11}$$

where $A_{nom}(q_e, \omega_e) = -\frac{1}{4}q_{ev}^T\omega_e\omega_e + \frac{1}{4}T(q_e)\omega_e^\times\omega_e - \frac{1}{2}T(q_e)J_0^{-1}(\omega_e + C\omega_d)^\times J_0(\omega_e + C\omega_d) + \frac{1}{2}T(q_e)(\omega_e^\times C\omega_e - C\omega_d)$, $B_{nom}(q_e, \omega_e) = \frac{1}{2}T(q_e)J_0^{-1}$, and $\bar{d}(t) \in \Re^3$ contains both the inertia uncertainty and external disturbances bounded lumped term.

Remark 1. From (11), it is obvious that first time the control input appeared into the second time derivative of sliding surface (8). Therefore, system relative degree $r = 2$.

Remark 2. (11) could be expressed as $\ddot{\sigma} \in [-C,\ C] + [K_m,\ K_M]u$, where the constants C, K_m, and K_M bounds are state dependent (further detail could be found in [20]).

Remark 3. In [26], second order sliding mode control has been designed. However, both the sliding mode order and relative degree are same and chattering is not eliminated.