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Filtered dynamic inversion for altitude control of fixed-wing unmanned air vehicles

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ABSTRACT

Instrumented unmanned air vehicles (UAVs) represent a new way of conducting atmospheric science, particularly within the atmospheric boundary layer where the air is turbulent. However, using autonomous UAVs for airborne measurement requires active control methods capable of following altitude commands despite unknown and turbulent disturbances to the air. Filtered dynamic inversion (FDI) is a control method with desirable command-following and disturbance-rejection properties for this application. FDI requires limited model information and is thus robust to parametric uncertainty, which arises in modeling UAV dynamics. In this paper, FDI is implemented in an altitude-flight-control system for an autonomous fixed-wing UAV. The control system is validated in simulation with a nonlinear dynamic model of a small fixed-wing UAV. The control system is also implemented and validated in flight experiments with turbulent wind conditions. Experimental results show that FDI yields improved altitude and pitch command following as compared to a classical (e.g., proportional-integral) flight-control system. In particular, experimental data demonstrate that the average power of the altitude and pitch command-following errors with FDI is smaller than those with proportional-integral control.

1. Introduction

Advances in sensor miniaturization have improved the viability of small unmanned air vehicles (UAVs) for a wide range of applications, including precision agriculture, search and rescue, and aerial surveillance. In addition to these vision-sensing applications, small UAVs can also be used to take meteorological measurements [1–5].

For several decades, manned aircraft have been used for atmospheric research such as conducting weather reconnaissance; measuring wind, temperature, and humidity profiles [6–8]; measuring atmospheric turbulence [9]; and tracking pollutant concentrations [10]. Small UAVs have advantages over manned aircraft, including reduced operational costs and the ability to operate and obtain measurements close to the Earth's surface [11]. Despite their potential, the use of UAVs for atmospheric research is still in its infancy, focusing on remotely piloted UAVs for obtaining wind, temperature, and humidity profiles [12,13]. Measurements during autonomous flight have been reported in Refs. [1,4,14–16].

Two of the traits, namely, small size and light weight, that are driving the increase in UAV usage for meteorological measurements also introduce challenges. Specifically, small lightweight UAVs are susceptible to the quasi-random forcing introduced by turbulence in the atmospheric boundary layer near the Earth's surface [17]. This boundary layer is the predominant region of interest for studying transport processes between the surface and the atmosphere. However, turbulence in the boundary layer can disturb a UAV's flight path and, thus, adversely impact the statistical accuracy of measurements from onboard sensors.

To improve the suitability of small lightweight UAVs as sensor platforms, it is necessary to improve their ability to correct for the impact of wind gusts induced by the turbulence. One approach is to improve the capabilities of the flight-control system. This paper examines the use of filtered dynamic inversion (FDI) for altitude control of a small fixed-wing UAV. FDI is a control method for highly uncertain minimum-phase linear dynamic systems, and is effective for command following in the presence of unmeasured disturbances [18,19]. In particular, Ref. [18] shows that for sufficiently large choice of a single control parameter, FDI makes the average power of the command-following error arbitrarily small despite unmeasured disturbances (e.g., turbulent wind). FDI is also effective for systems with nonlinear dynamics [20]. In this paper, an FDI control system is designed and implemented on a small fixed-wing UAV to achieve effective altitude command following in the presence of turbulent wind.

The main contributions of this paper include the design of an FDI flight-control system for a small fixed-wing UAV, and the

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Nomenclature

Filtered dynamic inversion

$A,\ B,\ C$	state-variable system matrices
x	state variable
u	control input
w	disturbance input
y	output
H_d	first nonzero Markov parameter
d	relative degree
$\mathbf{p} \triangleq \mathrm{d}/\mathrm{d}t$	differential operator
$\alpha_{\rm m},\ \beta_{\rm m}$	reference-model polynomial matrices
r	reference-model input
$y_{\rm m}$	reference-model output
$z \triangleq y - y_{\rm m}$	error
$\mathcal{P}_z$	average power of z
u_*	ideal dynamic inversion control
ρ	FDI controller order
k	FDI parameter
η_k	FDI polynomial

Nonlinear UAV dynamics

$F_{\rm I}$	inertial frame	
$o_{\rm I}$	center of $F_{\rm I}$	
$\hat{\imath}_{\rm I},\ \hat{\jmath}_{\rm I},\ \hat{k}_{\rm I}$	orthogonal unit vectors of $F_{\rm I}$	
$F_{\rm B}$	body frame	
$o_{\rm B}$	center of $F_{\rm B}$, which is the center of mass	
$\hat{\imath}_{\rm B},\ \hat{\jmath}_{\rm B},\ \hat{k}_{\rm B}$	orthogonal unit vectors of $F_{\rm B}$	
$\vec{r}$	position of $o_{\rm B}$ relative to $o_{\rm I}$	m
$X,\ Y,\ Z$	components of $\vec{r}$ resolved in $F_{\rm I}$	m
$\vec{v}$	velocity of $o_{\rm B}$ relative to $o_{\rm I}$ with respect to $F_{\rm I}$.	m/s
$v_{\rm B}$	$\vec{v}$ resolved in $F_{\rm B}$	m/s
$U,\ V,\ W$	components of $v_{\rm B}$	m/s
$\vec{\omega}$	angular velocity of $F_{\rm B}$ relative to $F_{\rm I}$	rad/s
$\omega_{\rm B}$	$\vec{\omega}$ resolved in $F_{\rm B}$	rad/s
$P,\ Q,\ R$	components of $\omega_{\rm B}$	rad/s
$\phi,\ \theta,\ \psi$	yaw, pitch, roll Euler angles	rad
m	mass	kg
$\vec{I}$	physical inertia matrix	$\mathrm{kg\,m^2}$
$I_{\rm xx},\ I_{\rm yy},\ I_{\rm zz}$	moments of inertia	$\mathrm{kg\,m^2}$
$I_{\rm xz}$	product of inertia	$\mathrm{kg\,m^2}$
$\vec{F}_{\rm a}$	aerodynamic force	N
$X_{\rm a},\ Y_{\rm a},\ Z_{\rm a}$	components of $\vec{F}_{\rm a}$ resolved in $F_{\rm B}$	N
$\vec{F}_{\rm T}$	thrust force	N
$X_{\rm T},\ Y_{\rm T},\ Z_{\rm T}$	components of $\vec{F}_{\rm T}$ resolved in $F_{\rm B}$	N
$\vec{g}$	acceleration due to gravity	$\mathrm{m/s^2}$
$\vec{M}_{\rm c}$	moment due to aerodynamic force	N m
$L,\ M,\ N$	components of $\vec{M}_{\rm c}$ resolved in $F_{\rm B}$	N m
$\vec{v}_{\rm w}$	wind velocity	m/s
$\vec{v}_{\rm r} \triangleq \vec{v} - \vec{v}_{\rm w}$	relative velocity	m/s
$U_{\rm r},\ V_{\rm r},\ W_{\rm r}$	components of $\vec{v}_{\rm r}$ resolved in $F_{\rm B}$	m/s
α	angle of attack	rad
β	sideslip angle	rad
$V_{\rm T}$	airspeed	m/s
$\delta_{\rm e}$	elevator deflection angle	rad
$\delta_{\rm r}$	rudder deflection angle	rad
$\delta_{\rm a}$	aileron deflection angle	rad

FDI flight control system

$h \triangleq -Z$	altitude	m
$h_{\rm d}$	altitude command	m
$U_{\rm d}$	speed command	m/s
$\theta_{\rm d}$	pitch command	rad
$u_{\rm T}$	throttle command	
$u_{\rm e}$	elevator command	rad
$\tilde{\theta} \triangleq \theta_{\rm d} - \theta$	pitch error	rad
$\mathcal{P}_h(t_1, t_0)$	average power of the altitude error on the $[t_0, t_1)$ time interval	$\mathrm{m^2}$
$\mathcal{P}_\theta(t_1, t_0)$	average power of the pitch error on the $[t_0, t_1)$ time interval	$\mathrm{rad^2}$
$k_{h,\rm p}$	altitude controller proportional gain	
$k_{h,\rm i}$	altitude controller integral gain	
$g_{\rm T}$	engine gain	
$k_{\rm T,p}$	speed controller proportional gain	
$k_{\rm T,i}$	speed controller integral gain	
C_k	transfer function for FDI pitch controller	

Linearized equations of motion for longitudinal flight

U_0	equilibrium $\hat{\imath}_{\rm B}$-direction velocity	m/s
W_0	equilibrium $\hat{k}_{\rm B}$-direction velocity	m/s
θ_0	equilibrium pitch	rad
$X_{\rm T,0}$	equilibrium $\hat{\imath}_{\rm B}$-direction thrust	N
$\delta_{\rm e,0}$	equilibrium elevator deflection	rad
ΔU	$\hat{\imath}_{\rm B}$-direction velocity perturbation	m/s
ΔW	$\hat{k}_{\rm B}$-direction velocity perturbation	m/s
$\Delta\theta$	pitch perturbation	rad
$\Delta X_{\rm T}$	$\hat{\imath}_{\rm B}$-direction thrust perturbation	N
$\Delta\delta_{\rm e}$	elevator deflection perturbation	rad
$x_{\rm l}$	state variable for longitudinal flight	
$A_{\rm l}$	dynamics matrix for longitudinal flight	
$B_{{\rm l},X_{\rm T}}$	$\Delta X_{\rm T}$ input matrix for longitudinal flight	
$B_{{\rm l},\delta_{\rm e}}$	$\Delta\delta_{\rm e}$ input matrix for longitudinal flight	
$\tau_{\rm e}$	elevator servomechanism time constant	s
$\Delta U_{\rm d}$	speed command perturbation	m/s
$\Delta u_{\rm e}$	elevator command perturbation	rad
G	transfer function for linearized longitudinal flight dynamics from $\Delta u_{\rm e}$ to $\Delta\theta$	
$\tilde{G}_k$	closed-loop transfer function for linearized dynamics from $\Delta u_{\rm e}$ to $\tilde{\theta}$ with FDI control	

Other parameters

$b_{\rm r}$	wing span	m
$c_{\rm r}$	mean cord length	m
$S_{\rm r}$	planform area	$\mathrm{m^2}$
$\rho_{\rm a}$	density of air	$\mathrm{kg/m^2}$
$k_{\theta,\rm p}$	baseline pitch controller proportional gain	
$k_{\theta,\rm i}$	baseline pitch controller integral gain	
$k_{\phi,\rm p}$	roll controller proportional gain	
$k_{\phi,\rm i}$	roll controller integral gain	
$T_{\rm s}$	sample time	s