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Robust and optimal attitude control of spacecraft with inertia uncertainties using minimal kinematic parameters

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ABSTRACT

In this paper a robust and optimal attitude control design that uses the minimal kinematic parameters and angular velocities feedback is presented for the three-axis attitude stabilization of spacecraft with inertia uncertainties. After proposing a new class of robust attitude control laws for the three-axis attitude stabilization of spacecraft with inertia uncertainties, it is shown that the proposed robust attitude control laws are optimal with respect to performance indices. A numerical example is given to illustrate the theoretical results presented in this paper.

1. Introduction

The robust attitude control problem of spacecraft is one of the important issues in the field of aerospace engineering because spacecraft is subject to parameter uncertainties and disturbances in orbit. The early results on the robust attitude control of rigid spacecraft have mainly used the unit quaternion for mathematically describing the spacecraft orientation (e.g., [1–7]). Among the results, Karray and Modi [4] proposed a robust attitude control design based on the variable structure control technique for the slewing and pointing maneuvers of spacecraft with parametric uncertainties. Brockman and Corless [5] developed necessary and sufficient conditions for quadratic boundedness of a class of nominally linear uncertain systems and applied their results to the attitude stabilization of the angular velocity subsystem of axisymmetric spacecraft subject to disturbance torques. Park [6] applied the inverse optimal control method of [8] to the attitude control of spacecraft with external disturbances. Later, Park [7] presented the inverse optimal and robust nonlinear attitude control law for the attitude control of spacecraft with control input uncertainties.

In this paper the problem of robust and optimal three-axis attitude stabilization of rigid spacecraft with inertia uncertainties is addressed. The complete attitude motion of rigid spacecraft, which includes dynamics and kinematics, described in terms of either the Cayley–Rodrigues parameters [9] or the Modified Rodrigues parameters [9,10] is considered. Both can be viewed as normalized versions of the unit quaternion and reduce the number of coordinates necessary to describe the kinematics from four to three by eliminating the unity length constraint associated with the unit quaternion. Hence, they are minimal and reduce the complexity of the kinematics, while the unit quaternion is non-minimal and is subject to the unity length constraint. Also, it is well-known that the mapping from the unit quaternion to the configuration space of the attitude motion, SO(3), is two to one. Note that SO(3) is defined by

$$\mathrm{SO}(3) = [A \in \mathrm{GL}(3, \Re) \mid A^T A = I_3, \det(A) = 1], \tag{1}$$

where $\mathrm{GL}(3, \Re)$ implies the linear group of all invertible 3×3 matrices with real coefficients, I_3 denotes the 3×3 identity matrix, and $\det(A)$ implies the determinant of the matrix A [11]. This feature of mapping results in a sign of ambiguity for any quaternion representation of a point in SO(3) and gives an alternative representation of the same point [12]. Nevertheless, the unit quaternion has been used as the kinematic parameter set with a popular choice.

There have been some remarkable studies for the attitude stabilization of rigid spacecraft using minimal, three-dimensional parameterizations for the kinematics (e.g., [13–17]). Slotine and Benedetto [13] proposed a proportional-derivative type of stabilizing control law using the Cayley–Rodrigues parameters for the attitude regulation of rigid spacecraft, and also derived an adaptive control law with the adaptation process of inertia matrix. Tsiotras [14] developed a class of linear stabilizing control laws for the attitude stabilization of rigid spacecraft with the kinematic description in terms of either the Cayley–Rodrigues parameters or the Modified Rodrigues parameters, and also derived a new kinematic parameter set that yields only two control torques. Also, Tsiotras [15]

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extended the result of [14] to the design of nonlinear control laws with scalar feedback gains, and investigated the optimality characteristics of the control laws. Krstić and Tsiotras [16] presented a design method yielding the optimal feedback control law for the attitude regulation of rigid spacecraft with the Cayley–Rodrigues parameters by using the inverse optimal control approach of [8] and [18]. Especially, Park et al. [17] first proposed the optimal fuzzy control law that has the robustness with respect to a class of input uncertainties for the attitude stabilization of rigid spacecraft with the kinematic description using the Cayley–Rodrigues parameters.

Though the studies of [15] and [16] result in well-established optimal stabilization designs for rigid spacecraft, both have a common drawback that the exact knowledge of the system parameters is required to adopt the control law in real application. In many practical situations, however, systems may have unknown parametric uncertainties and, therefore, their designs may not be adopted in practice. A design method that may overcome this problem can be found in [17], where a fuzzy control method that does not require the exact system parameters is utilized to design the optimal control law for the attitude stabilization of rigid spacecraft. But, the design of [17] yields undesirable high control gains, which is caused by setting the penalty function on the control input to be identity matrix to guarantee a robustness property of the controller, and does not give the rigorous proof for the robustness property with respect to the system parametric uncertainties. Also, the designs of [16] and [17] have cost functions depending on the particular system dynamics, which is caused by their particular controller structures, and this restricts the design of optimal controller that has a tolerance in choice of the feedback gain structure. Thus, an alternative design method to consider the robustness as well as the optimality in performance with a tolerance in design of the feedback gain structure may be needed, which is the main motivation of this paper.

With the above motivation, in this paper a new class of robust and optimal control laws using minimal kinematic parameters and positive definite gain matrices is presented for the optimal attitude stabilization of rigid spacecraft with inertia uncertainties. Global asymptotic stability of the proposed control laws is shown by using the LaSalle Invariance Principle [19]. The present paper may be viewed as extending the result of [3], where the unit quaternion is used for describing the kinematic equations. Joshi et al. [3] addressed the conditions for the existence of the closed-loop equilibrium solutions by using the known control law that was already stated by [20], and gave the explicit stability proof of the closed-loop system. Besides the different kinematic parameterizations, the proposed method guarantees global asymptotic stability of the closed-loop system for any choice of positive definite gain matrices, while the study of [3] guarantees only local asymptotic stability for such cases. Also, the optimality properties of the proposed control laws are provided by using the Hamilton–Jacobi theory [21].

This paper is organized as follows: First, the equations of rigid body motion with inertia uncertainties are given. Next, a new class of robust control laws is proposed for the attitude stabilization of the rigid body motion with inertia uncertainties. Also, the optimality properties of the proposed control laws are investigated. Then, a numerical example is given to illustrate the theoretical results presented in this paper. Finally, this paper is concluded with some remarks.

2. Definition of symbols

The symbols used in this paper are defined as follows:

$J_n \in \Re^{3\times3}$	Nominal value of the inertia matrix.
$\Delta J \in \Re^{3\times3}$	Inertia matrix uncertainty.
$J \in \Re^{3\times3}$	$J_n + \Delta J$, Inertia matrix.
$\omega \in \Re^3$	Body angular velocity vector in a body-fixed frame.
$u \in \Re^3$	Control torque vector.
$S(\omega) \in \Re^{3\times3}$	$\begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix}$.
$\hat{e} \in \Re^3$	Euler axis.
$\phi \in \Re$	Euler angle.
$\rho \in \Re^3$	$\hat{e}\tan\left(\frac{\phi}{2}\right)$, Cayley–Rodrigues parameters.
$\sigma \in \Re^3$	$\hat{e}\tan\left(\frac{\phi}{4}\right)$, Modified Rodrigues parameters.
$I_3 \in \Re^{3\times3}$	3×3 identity matrix.
$H(\rho) \in \Re^{3\times3}$	$\frac{1}{2}[I_3 - S(\rho) + \rho\rho^T]$.
$G(\sigma) \in \Re^{3\times3}$	$\frac{1}{2}\left[\left(\frac{1-\sigma^T\sigma}{2}\right)I_3 - S(\sigma) + \sigma\sigma^T\right]$.
diag	Diagonal matrix.
$\|x\|$	Euclidean norm of x.
$\lambda_{\max}(J)$	Maximum eigenvalue of J.

3. Rigid body model with inertia uncertainties

The dynamics of the rotational motion of the rigid body with inertia uncertainties are described by the following set of differential equations [22]:

$$(J_n + \Delta J)\dot{\omega} = S(\omega)(J_n + \Delta J)\omega + u, \ \omega(0) = \omega_0. \tag{2}$$

In this paper the dynamics of the rigid body orientation with respect to the inertia frame is given in terms of either the Cayley–Rodrigues parameters [9] or the Modified Rodrigues parameters [9,10]. The kinematic equations in terms of the Cayley–Rodrigues parameters take the form [9]

$$\dot{\rho} = H(\rho)\omega, \qquad \rho(0) = \rho_0. \tag{3}$$

Also, the kinematic equations in terms of the Modified Rodrigues parameters take the form [9]

$$\dot{\sigma} = G(\sigma)\omega, \qquad \sigma(0) = \sigma_0. \tag{4}$$

Note that $H(\rho)$ in (3) and $G(\sigma)$ in (4) have the following property

$$\rho^T H(\rho) = \frac{1}{2}(1 + \rho^T\rho)\rho^T, \ \ \forall \rho \in \Re^3 \tag{5}$$

and

$$\sigma^T G(\sigma) = \frac{1}{4}(1 + \sigma^T\sigma)\sigma^T, \ \ \forall \sigma \in \Re^3, \tag{6}$$

respectively [15]. The kinematic description using the Cayley–Rodrigues parameters can describe eigenaxis rotations up to 180 deg, whereas the Modified Rodrigues parameters remains valid for eigenaxis rotations up to 360 deg [9,15]. Note that, however, the possible singular configurations corresponding to the body orientation can be avoided by applying any control law over an arbitrarily short period of time to move the body away from the singular configuration [15].

4. Main results

4.1. Robust attitude stabilization of rigid body

In this section a new class of robust stabilizing control laws is proposed for the two cases of the complete attitude motion of rigid spacecraft with inertia uncertainties.