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An inverse trigonometric shear deformation theory for supersonic flutter characteristics of multilayered composite plates

Neeraj Grover [a], B.N. Singh [b,*], D.K. Maiti [b]

[a] Department of Mechanical Engineering, Thapar University, Patiala, Punjab, 147 004, India
[b] Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur, West Bengal 721 302, India

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ABSTRACT

In this study, an inverse trigonometric shear deformation theory developed by the authors is extended to assess the flutter behavior of multilayered composite plates subjected to yawed supersonic flow. The shear deformation is considered in terms of an inverse cotangent function which yields non-linear distribution of shear stresses. A generalized finite element formulation is presented to consider the shear strain function based theories. The displacement field is modified by a precise involvement of additional field variables to ensure the implementation of C^0 continuous finite element. First order piston theory is employed to consider the aerodynamic load. The applicability, validity and accuracy of the present mathematical treatment are ascertained by performing various numerical tests and comparing the present results with the existing results. The influences of various parameters such as lamination sequences, boundary conditions, material anisotropy, flow angles, etc. on the free vibration and flutter behavior are examined and significant conclusions are made. It is concluded that flow angles, lamination sequence and material anisotropy should be considered as essential design parameters for enhanced flutter boundary of supersonic vehicles.

1. Introduction

The composite materials are increasingly being employed in the design of structures due to their enhanced mechanical properties such as specific strength, specific stiffness, ability for the tailor made designs, resistance to corrosion, and response to the environmental effects as compared to monolithic materials. The most widespread use of composites continues to be for aircraft structural components for both the airframe and the engine components. Indeed for the airframe it is anticipated that a weight saving in the range 30%–70% may be achieved through the use of composites. The increased usage of composite material in the structural applications is in the form of the layered structures. There had been significant attention in the past towards the modeling of the layered structures. The complicating effects such as shear deformation, zig-zag requirement and interlaminar continuity were dealt in the past [1]. Depending upon these factors, there exist various shear deformation theories which consider the shear deformation in a unique way. Reissner [2] and Mindlin [3] considered the linear shear deformation theory which is termed as first order shear deformation theory (FSDT). The FSDT doesn't account for the zero transverse shear conditions on top and bottom surfaces of the plate and therefore a shear correction factor (SCF) is required. However, it was shown that the value of SCF depends upon the lamination sequence, boundary conditions, etc. [4] and hence FSDT is less realistic. In order to eliminate the requirement of SCF and properly address the shear deformation, a number of higher order shear deformation theories (HSDT) were developed. The development of HSDTs can be observed in the form of polynomial shear deformation theories (PSDTs) and non-polynomial shear deformation theories (NPSDTs). The higher order terms in PSDTs [5–9] are expressed by means of Taylor's expansion while in case of NPSDTs, a function of thickness co-ordinate is chosen to represent the realistic shear deformation [10–25]. The accurate mathematical modeling of the layered structures is essential since it is the prime factor which determines the accuracy of the structural kinematics of the composite and sandwich plates. The significant reviews on the development of various theories are given in Refs. [1,26–34].

Most of the aircraft and space vehicle structures, where composites have tremendous applications, are subjected to severe aerodynamic loads during their flight. These aerodynamic forces influence the static and dynamic response of the composite structures. There are instances where these structures loose their stability due to the interaction of aerodynamic, elastic and inertial forces. The

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[*] Corresponding author. Tel.: +91 3222 283026.
E-mail address: bnsingh@aero.iitkgp.ernet.in (B.N. Singh).

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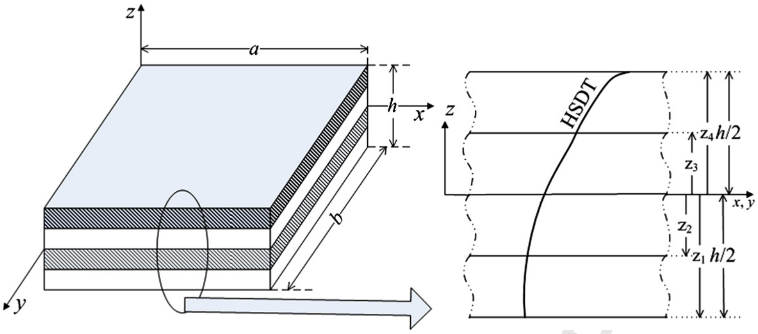

Fig. 1. Plate kinematics in Cartesian co-ordinate system.

self excited oscillation of the external skin of the aircraft due to such interactions is called flutter. In order to ensure the safe operational conditions corresponding to flutter, it is desired to know the flutter boundary which is defined in terms of critical dynamic pressure and critical frequency parameter. In the earlier developments, the behavior of flutter was investigated by Fung [35], and Dowell [36,37]. The flutter studies belonging to isotropic and laminated composites structures were reviewed by Dowell [38], Bismark [39,40] and Mei et al. [41]. Sander et al. [42] employed a C^0 conforming quadrilateral finite element based on Kirchhoff plate theory and utilized linear piston theory to examine the supersonic flutter behavior of isotropic plates. The free vibration and flutter behavior of laminated quadrilateral plates assuming classical laminated plate theory (CLPT) and linear piston theory aerodynamics was examined by Srinivasan and Babu [43,44]. The supersonic flutter behavior of composite skew plates was studied by Chowdary et al. [45,46]. They used FSDT to consider the structural kinematics of the plate and linear piston theory for the aerodynamic forces. Olson [47] employed the finite element analysis to examine flutter modes and frequencies. Ganapathi and Patel [48] implemented a linear piston theory in conjunction with finite element to study the supersonic flutter behavior of laminated composites. Nam and Hwang [49] studied the effects of hysteresis on the supersonic flutter characteristic implementing linear piston theory. Various parametric studies [50–52] have been performed for the flutter behavior of composite and functionally graded plates. A scheme for the enhancement of flutter velocity of composites was presented by Raja [53]. They implemented direct matrix abstraction programme (DMAP) of NASTRAN and considered subsonic aerodynamics through doublet lattice method. Mahato and Maiti [54,55] studied the aeroelastic performances of smart composites in hygro-thermal environment implementing subsonic aerodynamics. It is observed that most of the studies conducted on the flutter analysis are performed using the CLPT or FSDT. However, it should be noted that the shear deformation effects are not considered in CLPT while FSDT requires the use of shear correction factor. Moreover, no attempt has been made to study the influence of the variation of the material properties on the critical dynamic pressure. The lamination sequence and the fiber direction in the composites influence its material properties and there may be substantial effects of these variations on the flutter behavior of composite plates.

In the present work, a newly developed inverse trigonometric shear deformation theory (ITSDT) in a recent work by the authors [56] is extended to model the structural kinematics of the plate and applied for the free vibration and flutter behavior of multilayered composite plates. The theory assumes the non-linear shear deformation in terms of an inverse trigonometric shear strain function and also satisfies the zero transverse shear conditions on top and bottom surfaces of the plate. Therefore, the theory considers the shear deformation in more realistic manner and SCF is no longer required. The field variables are elegantly utilized to ensure the C^0 continuity requirement. Linear piston theory is implemented to consider the aerodynamic load which is applicable for the flow regimes corresponding to Mach, $M > 1.7$. Free vibration and flutter behavior of isotropic and multilayered composite plates are investigated in terms of frequencies and dynamic pressure parameters. The ability of ITSDT to predict higher modes of vibration is ensured by computation of these modes for a few cases. The effects of boundary conditions, lamination sequence, flow angularity, and material anisotropy on the flutter behavior of isotropic and laminated composite plates are examined and the results are compared with the published results whichever available. Some new results of flutter behavior are also presented in this work.

2. Mathematical formulation

The plate under consideration is a layered plate constituted of the orthotropic layers. The schematic and dimensions $(a \times b \times h)$ of a general laminated plate are shown in Fig. 1 in a Cartesian coordinate system $(x–y–z)$.

2.1. Structural kinematics

The considered multilayered composite plates are modeled by utilizing an inverse trigonometric shear deformation theory [56]. The displacement field is expressed in terms of mid-plane displacements (u_0, v_0, and w_0), mid-plane rotations (θ_x and θ_y) and a shear strain function of thickness co-ordinate termed as $g(z)$ which describes the non-linear shear deformation. The displacement field for the considered plates is as follow:

$$\begin{aligned}
u(x, y, z) &= u_0(x, y) - z\frac{\partial w_0}{\partial x} + \big(g(z) + \Omega z\big)\theta_x(x, y) \\
v(x, y, z) &= v_0(x, y) - z\frac{\partial w_0}{\partial y} + \big(g(z) + \Omega z\big)\theta_y(x, y) \\
w(x, y, z) &= w_0(x, y) \qquad (1)
\end{aligned}$$

Here, the shear strain function $g(z) = \cot^{-1}(rh/z)$ ensures the nonlinear shear deformation. The parameter r is the shape parameter whose value is ascertained as 0.46 by comparing the present results with the exact solution in the post processing step [56]. The value of the parameter Ω is evaluated after satisfying the zero transverse shear conditions on the top and bottom surfaces of the plate and is equal to $-d/dz(g(z))|_{z=\pm\frac{h}{2}}$. The finite element implementation of the above mentioned displacement field requires a C^1 continuous element due to the presence of derivatives of the transverse deflection (w_0) in the in-plane displacements (u, v). However, in the present work, additional degrees of freedom $\phi_x = \frac{\partial w_0}{\partial x}, \phi_y = \frac{\partial w_0}{\partial y}$ are adequately imposed in order to restrict the continuity requirement to C^0. These additional degrees of freedom may contribute towards the strain energy which is taken into consideration using the variations [9] as indicated in Eq. (13). The modified displacement field is then written as:

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