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Improved zigzag theories for laminated composite and sandwich plates with interlaminar shear stress continuity

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A B S T R A C T

In this present work, improved zigzag theories are developed for the flexural analysis of laminated plates using algebraic, hyperbolic, inverse trigonometric and trigonometric shear strain functions. The governing differential equations and boundary conditions of the structural system are obtained through the principle of virtual work. A generalized Navier closed form solution technique is applied for the flexural analysis of laminated plates. The present theories fulfill the transverse shear stress continuity and in-plane displacement continuity at each layer interfaces. Moreover, the present theories exhibit a constant variation of transverse displacement and parabolic variation of transverse shear stresses across the plate thickness. The tangential stress free boundary conditions are satisfied on the external surfaces of the panel; hence the necessity of artificial shear correction factor is ignored. The present theories consist of 5 unknowns as in the case of FSDT. Several numerical examples are carried out for a broad range of lamination sequence and geometric parameters. To reveal the potency and performance of the present models, numerical comparisons are made with the 3D elasticity solution and other numerical methods and it is observed that the present models perform very well for the static behavior of laminated plates.

1. Introduction

Multilayered reinforced composite structures possess highly desirable mechanical features such as high strength to weight ratio, better fatigue strength, better resistance to corrosion and design flexibility. As a result, reinforced composite structures are continuously increasing in aerospace, automobile, civil, marine and many other industries. Notably, advanced composite materials such as glass-fiber, carbon-fiber and boron-fiber were used as a part of aircraft structures during World War II. Currently, the contribution of composite materials augmented in airborne vehicles such as military aircrafts, commercial aircrafts, gliders and helicopters. Specifically, composite materials are widely used in wing-fuselage fairings, control surfaces, leading and trailing edges of wing panels, engine pylon-fairings, engine cowling and rotor blades. In addition, complete structures being constructed by composite materials for modern gliders [1]. For instance, over 50% of structural components of commercial aircrafts Boeing 787 and Airbus 350XWB are composed of composite materials than isotropic materials [2]. Military airplanes (F-22, F-35 and F-117A) also widely used the composite materials.

Multilayered structures (flat plate and curved shells) composed of N number of layers that are perfectly bonded together and which can be made of isotropic, orthotropic, as well as anisotropic materials. Laminated structures possess high value of in-plane Young's modulus ratio ($E_1/E_2 = 5$–40) to transverse shear modulus ratio ($G_{12}/E_2 = 0.1$–0.005). Hence, laminated structures become weak at the interlaminar shear strength than the conventional materials. As per two dimensional modeling (2D) concern, fulfillment of C^0 requirement (interlaminar shear stress continuity and zigzag form of in-plane displacement continuity) of multilayered structure is a cumbersome subject. Thus an efficient and reliable mathematical model needs to be developed in order to accurately address the C^0 requirement of multilayered plates. Hence, a large number of shear deformation plate theories have been developed in the past few years, to predict them efficiently. Classical laminated plate theory [3] not considered the transverse shear effects, thus it is inadequate for sandwich plates. First order shear deformation theory [4–6] (FSDT) is the extension of Mindlin [7] and Reissner [8] which assumes a linear variation of transverse shear strains through the plate thickness. Therefore an artificial shear correction factor has to be considered.

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Notations

a, b, h	length, width and thickness of the laminate
x, y, z	Cartesian coordinate of the laminate
θ	fiber orientation angle
N	total number of layers in the laminate
k	respective layer
U^k, V^k	in-plane displacement components of kth layer
W	transverse displacement component
u_0, v_0, w_0	mid-plane displacements
ϕ_x, ϕ_y	mid-plane rotational deformations
E_1, E_2, E_3	Young's moduli
G_{12}, G_{23}, G_{13}	Shear moduli
$\nu_{12}, \nu_{13}, \nu_{23}$	Poisson's ratios
$\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \tau_{yz}$	stresses at a point
m	transverse shear stress parameter
$f(z)$	transverse shear strain shape function
A^k, B^k, C^k, D^k	zigzag parameters
q	transverse load

To overcome the above confines higher order shear deformation theories (HSDT) were developed. Basset [9] introduced the displacement field in terms of the Taylor series expansion of thickness coordinate. Lo et al. [10] have developed a higher order plate theory for homogeneous plates. Reddy [11] has given a theory, which gives parabolic distribution of transverse shear stresses and improved in-plane stresses than the FSDT. Ambartsumian [12] presented a model for anisotropic plates and shells. Marur and Kant [13] have developed a theory for a laminated beams with third and second order of thickness coordinate in the in-plane displacements and transverse displacement respectively. Swaminathan and Patil [14] have developed a higher order plate theory with 12 unknowns. Matsunaga [15] presented a mathematical model which contains ninth and eighth order of thickness coordinates in the in-plane displacement and transverse displacement respectively. Levinson [16] and Murthy [17] have presented a cubic order shear deformation theory which makes zero transverse shear stresses at the upper and lower surface of the plate.

Levy [18] introduced a sinus trigonometric shear deformation theory. Moreover, remarkable works based on various shear strain function can be found in [19–27]. The above mentioned theories represents a nonlinear variation of transverse shear stresses and achieves the traction free boundary conditions. However, these single layer theories represent a continuous shear strain variation, which leads to transverse shear stress discontinuity at the interfaces. Consequently, various researchers have focused to develop efficient mathematical models to predict the geometric continuity (GC) and inter-laminar stress continuity (IC).

Carrera [28] studied a mixed layerwise theory in which Legendre polynomials are handled and accurate evaluation of the structural responses is predicted. A layerwise model given by Srinivas [29] in which the in-plane displacement components are considered to be piecewise linear whereas normal displacement was taken as constant. Toledano and Murakami [30] have used the Reissner mixed variational principle in order to ensure the IC and they considered the piecewise linear functions in the in-plane displacement component. Reddy et al. [6] discussed a two dimensional layerwise theory, where the layerwise expansions are involved in the in-plane and transverse displacement components. Ambartsumyan [31] has proposed a theory for homogeneous materials with IC and later on this theory has been refined by numerous researchers. A discrete layerwise theory provided by Cho et al. [32]. To improve the dynamic response of laminated composite plates they assumed a third and second order of thickness-coordinate in the in-plane and normal displacement components correspondingly. Ferreira [33] given a layerwise theory in which the differential equation and the boundary conditions are derived using Radial basis function (RBF). The above listed discrete layerwise theories predict the zigzag requirement (ZZ) and transverse shear stress continuity at high computational cost. Because the unknowns are strongly dependent on the layer increment. As a result, several researchers were motivated towards the development of zigzag theories.

In zigzag theory, the unknowns are taken in each interfaces intrems of those at the midplane. Static and dynamic analysis of laminated composite plates were analyzed using a piecewise linear displacement field attempted by Di Sciuva [34]. Various structural responses are predicted for symmetric and unsymmetric laminated plates using a plate theory of Whitney [35], which was the extension work of Ambartsumyan [31]. Ren theory [36] which allows the in-plane displacement and transverse shear stress continuity for the cross-ply laminated composite plates. Bhaskar and Varadan [37] studied the transverse shear deformation effects for laminated plates with layer independent unknowns. They obtained the transverse displacement and normal strain/stresses with adequate accuracy. Later Cho and Parmerter [38] have given a model where the in-plane displacement components consist of the cubic order of thickness coordinate with Heaviside step function. Icardi [39] used the third order zigzag model of Di Sciuva and Icardi [40] for curvelinear plate using an eight noded element with 56 unknown variables. A layerwise higher order zigzag theory (HOZT) presented by Lee et al. [41], assumes a cubic variation of in-plane displacement and parabolic variation of transverse shear stresses across the plate thickness. Carrera et al. [42] presented a model with zigzag functions which fulfills the ZZ effects at the interfaces. Demasi [43] given a shear deformation theory with the piecewise linear Murakami's zigzag functions using finite element method (FEM) for multilayered plates. Sheikh and Chakrabarti [44] have made an attempt on cubic order zigzag theory using six-noded non-conformity element. They achieved the GC and equilibrium condition of transverse shear stresses at the interfaces. Chalak et al. [45] have presented a cubic order zigzag theory with Heavy side step function. The transverse displacement field designed in such a way which assumes quadratic and linear variation of core and face respectively. Kapuria et al. [46] have presented a HOZT for the dynamic analysis of highly anisotropic laminated beams with damping and they successfully achieved the tangential shear stress boundary conditions and IC. Further, the same authors developed a coupled zigzag theory [47] for the evaluation of the static responses of piezoelectric sandwich beams. Lo et al. [48] studied the structural behaviors using a global–local higher-order theory in hygrothermal environment. Though, the above noted polynomial zigzag theories are layer independent, interpreting the higher order terms in the formulation is quite difficult.

So to avoid the above difficulties non-polynomial zigzag theories have been developed [49–52,22,53–56]. They allow the ZZ requirement and IC with easy formulation and enhanced results. Also, the shear strain function makes the zero transverse shear stresses at the top and bottom surface of the plate a priori. Shimpi and Ghugal [49] have introduced a layerwise shear deformation theory with involving trigonometric shear strain function for a laminated beams. Later Shimpi and Ghugal [50] have improved the work of Shimpi and Ghugal [49]. Again overlooked Refs. [50] for