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Solving the maximum-crossrange problem via successive second-order cone programming with a line search

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ABSTRACT

The maximum-crossrange problem is an optimal control problem of computing the maximum crossrange reachable by a hypersonic entry vehicle at a specified downrange, which has long known to be very difficult to solve due to its high nonlinearities and non-convexity. This paper presents how to convexify the problem so that it can be efficiently solved by successive second-order cone programming (SOCP). Particular focus is given on equivalent transformation of the original optimization objective and rigorous establishment of validity of the relaxation process used for convexification. In addition, it is observed that iteratively solving the SOCP problems may not always guarantee convergence to the original problem, a simple line search approach is proposed which is found critical to ensure the convergence of the successive SOCP method. Numerical demonstrations are provided to illustrate the effectiveness and efficiency of the proposed method and its applicability to both orbital and suborbital missions.

1. Introduction

In hypersonic lifting entry flight, a vehicle returning to the Earth from orbit or a burnout condition of a rocket, will continuously dissipate its energy along the gliding trajectory until a final state at a specified energy is reached. Then, terminal area energy management [1] is activated to prepare for the approach and landing of a reusable launch vehicle, or terminal guidance [2] is triggered. For a given amount of energy dissipation, the vehicle can cover certain downrange and crossrange. Determining the largest reachable area, also known as the landing footprint, is critical for mission planning of selecting a feasible landing site or target. To get the footprint, we can repeatedly solve the maximum-crossrange problem, which is defined to maximize crossrange when downrange is specified, for a series of prescribed downranges. Nevertheless, the maximum-crossrange problem is very difficult to solve due to high sensitivity and nonlinearities in the dynamics and terminal constraints. Path constraints such as maximum heating rate and dynamic pressure further compound the difficulty. Hence much of the existing work has primarily focused on approximating the footprint by making assumptions to simplify the computation.

Early work assumes a non-rotating Earth and an equilibrium glide condition to simplify the entry dynamics, and uses a scanning technique to approximate a footprint. The footprint is obtained by analytically solving a series of downrange-free maximum-crossrange problems (downrange is not specified) without consideration of path constraints [3–5]. The maximum-crossrange problem with full-state dynamics and path constraints has been numerically solved by a Legendre pseudospectral method [6,7]. Another analytic method to quickly generate a footprint is to solve a series of simple closest-approach problems based on a quasi-equilibrium glide condition, and this condition is used to convert path constraints into bank-angle constraints [8]. In Ref. [9], varying drag profiles observing the path constraints can be flown to approximate the footprint in real-time.

In this paper, we will solve the maximum-crossrange problem directly by a successive method. This method is to iteratively solve a sequence of second-order cone programming (SOCP) problems until the solutions converge to the solution of the original problem. One critical feature of this method is that it utilizes the complete entry dynamics. Hence, it is applicable to suborbital/abort missions and high L/D vehicles where the widely used equilibrium-glide approximation is not always valid. Moreover, the method can efficiently solve the problem because the SOCP problem in each iteration is guaranteed to be solved in polynomial time by an interior-point method whenever the problem is feasible [10–13].

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Numerical evidence in this work will demonstrate that the SOCP-based method obtains the solution significantly faster and more reliably than a general-purpose trajectory optimization software. This provides a distinct advantage since it enables quick checking of reachability of one/multiple potential targets without computing the entire footprint.

To apply successive SOCP, we first present how to convexify the original nonconvex problem into an SOCP formulation, particularly on convexification of the nonlinear performance index, entry dynamics, and terminal constraints. Relaxation is a common necessary technique for convexification, which involves enlarging admissible set of the original problem. Successful convexification should render the relaxed problem to have the same solution as the original problem. Ensuring validity of the relaxation is crucial but challenging. We will prove that the relaxation is indeed valid by applying a maximum principle in optimal control theory to the relaxed problem.

Next, we will iteratively solve the resulting SOCP formulation to approach the solution of the original problem. In general, proving convergence of this successive solution procedure is still an open challenge. When the SOCP formulation only involves linearization of concave inequality constraints, convergence is theoretically guaranteed [14]. Nevertheless, for complex systems with highly nonlinear dynamics and strict path constraints, convergence largely relies on a careful SOCP formulation of the original problem, such as the promising application of successive SOCP in a few practical engineering problems in recent years [15–17]. But, one notable observation in this paper is that the successive SOCP method may not converge for the maximum-crossrange problem. To ensure convergence, we propose a simple line search to make appropriate progress based on the constraint violation in each iteration. Effectiveness of this strategy in successive SOCP will be demonstrated by a number of numerical examples.

2. Problem formulation

In this section, we present formulation of the maximum-crossrange problem including a nonlinear performance index, entry dynamics, and all path and terminal constraints.

2.1. Entry dynamics and path constraints

The dimensionless equations of motion of an entry vehicle over a spherical rotating Earth with respect to energy are [16]

$$\begin{aligned}
r' &= (1/D)\sin\gamma \\
\theta' &= \cos\gamma\sin\psi/(rD\cos\phi) \\
\phi' &= \cos\gamma\cos\psi/(rD) \\
\gamma' &= [L\cos\sigma + (V^2 - 1/r)\cos\gamma/r + 2\Omega V\cos\phi\sin\psi \\
&\quad + \Omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\cos\psi\sin\phi)]/(V^2 D) \\
\psi' &= [L\sin\sigma/\cos\gamma + (V^2/r)\cos\gamma\sin\psi\tan\phi \\
&\quad - 2\Omega V(\tan\gamma\cos\psi\cos\phi - \sin\phi) \\
&\quad + \Omega^2 r\sin\psi\sin\phi\cos\phi/\cos\gamma]/(V^2 D)
\end{aligned} \tag{1}$$

where r is the radial distance from the Earth's center to the vehicle, θ and ϕ are the longitude and latitude respectively, V is the Earth-relative velocity, γ is the relative flight path angle, and ψ is the heading angle measured clockwise from the north. The variables r and V are scaled by R_0 and $\sqrt{g_0 R_0}$ respectively, where R_0 is the Earth's radius and g_0 is the Earth gravitational acceleration at R_0. The Earth self-rotation rate Ω is scaled by $\sqrt{g_0/R_0}$. The terms L and D are dimensionless aerodynamic lift and drag accelerations in g_0, that is $L = 0.5R_0\rho V^2 S_{\text{ref}} C_L/m$ and $D = 0.5R_0\rho V^2 S_{\text{ref}} C_D/m$, where ρ is the atmospheric density, S_{ref} is the reference area, m is the vehicle mass, and C_L (C_D) is the lift (drag) coefficient. The velocity, which differential equation is not included in Eq. (1), can be obtained by $V = \sqrt{2(1/r - e)}$, where the dimensionless energy e is used as the independent variable in Eq. (1).

In this paper we assume a given angle-of-attack profile, as in Ref. [8], and the bank angle σ, defined to be positive for banking to the right, is used to shape the entry trajectory. The bank-angle magnitude is generally bounded as follows

$$\sigma_{\min} \le |\sigma| \le \sigma_{\max} \tag{2}$$

where $\sigma_{\min}$ and $\sigma_{\max}$ are the lower bound and upper bound, respectively, and $0 \le \sigma_{\min} \le \sigma_{\max} \le 90°$. The above constraints are nonconvex if $\sigma_{\min} > 0$.

Path constraints on maximum allowed heating rate $\dot{Q}$, dynamic pressure q, and normal load n are given by

$$\dot{Q} = k_Q \sqrt{g_0 R_0}^{\,3.15} \sqrt{\rho}\, V^{3.15} \le \dot{Q}_{\max} \tag{3}$$

$$q = 0.5\, g_0 R_0\, \rho V^2 \le q_{\max} \tag{4}$$

$$n = \sqrt{L^2 + D^2} \le n_{\max} \tag{5}$$

where k_Q is a constant, $\dot{Q}_{\max}$, $q_{\max}$, and $n_{\max}$ are all dimensional with units of W/m^2, N/m^2, and g_0, respectively, and ρ, a function of r, has a unit of kg/m^3. Note that all path constraints are actually functions of r and e. At each e, we can find a minimum altitude numerically by intersection of the active path constraints; namely, the path constraints can be equivalently converted into

$$r(e) \ge l_1(e) \tag{6}$$

where l_1 is a lower bound on r. We can also impose an energy-dependent upper bound on r to make the vehicle always under sufficient aerodynamic control [17], i.e.,

$$r(e) \le l_2(e) \tag{7}$$

where l_2 could be determined by requiring $D \ge D_{\min}$.

2.2. Terminal constraints and performance index

First, terminal constraints include those on final altitude and flight path angle, i.e.,

$$r(e_f) = r_f^*, \quad \gamma(e_f) = \gamma_f^* \tag{8}$$

Note that the terminal flight path angle can also be limited in a range without loss of generality, i.e., $\gamma(e_f) \in [\gamma_{f,\min}, \gamma_{f,\max}]$. The final energy e_f is computed by $e_f = 1/r_f^* - (V_f^*)^2/2$, where V_f^* is the required final velocity.

Another terminal constraint is related to the final longitude and latitude. Let us see Fig. 1 where point $O(\theta_0, \phi_0)$ indicates the initial location of the vehicle, point $P(\theta_P, \phi_P)$ is a point along the initial heading direction ψ_0 at O, and point $F(\theta_f, \phi_f)$ is the final location of the vehicle. The maximum-crossrange problem requires $\angle OPF = 90°$, which results in the following right-angle constraint

$$\cos(S_{OF}) = \cos(S_{OP})\cos(S_{PF}) \tag{9}$$

where S_{OP} is the specified downrange and S_{PF} is the crossrange that needs to be maximized. For given S_{OP}, point P has the following coordinate

$$\phi_P = \sin^{-1}(\sin\phi_0\cos(S_{OP}) + \cos\phi_0\sin(S_{OP})\cos\psi_0)$$

$$\theta_P = \theta_0 + \text{atan2}(\sin\psi_0\sin(S_{OP})\cos\phi_0, \cos(S_{OP}) - \sin\phi_0\sin\phi_P)$$

where atan2 is a four-quadrant inverse tangent function, and