Reliability Engineering and System Safety 95 (2010) 887–896

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Bi and tri-objective optimization in the deterministic network interdiction problem

Claudio M. Rocco S. [a], José Emmanuel Ramirez-Marquez [b,*], Daniel E. Salazar A. [c]

[a] Facultad de Ingeniería, Universidad Central de Venezuela, Caracas, Venezuela
[b] Stevens Institute of Technology, Castle Point on Hudson, Hoboken, NJ 07030, USA
[c] CEANI-SIANI, Universidad Las Palmas de Gran Canaria, Spain

ARTICLE INFO

Article history:
Received 29 October 2009
Received in revised form
12 March 2010
Accepted 21 March 2010
Available online 1 April 2010

Keywords:
Network
Interdiction
Evolutionary
Optimization
Multi-objective

ABSTRACT

Solution approaches to the deterministic network interdiction problem have previously been developed for optimizing a single figure-of-merit of the network configuration (i.e. flow that can be transmitted between a source node and a sink node for a fixed network design) under constraints related to limited amount of resources available to interdict network links. These approaches work under the assumption that: (1) nominal capacity of each link is completely reduced when interdicted and (2) there is a single criterion to optimize. This paper presents a newly developed evolutionary algorithm that for the first time allows solving multi-objective optimization models for the design of network interdiction strategies that take into account a variety of figures-of-merit. The algorithm provides an approximation to the optimal Pareto frontier using: (a) techniques in Monte Carlo simulation to generate potential network interdiction strategies, (b) graph theory to analyze strategies' maximum source–sink flow and (c) an evolutionary search that is driven by the probability that a link will belong to the optimal Pareto set. Examples for different sizes of networks and network behavior are used throughout the paper to illustrate and validate the approach.

1. Introduction

The classical deterministic network interdiction problem (DNIP) can be viewed as a resource allocation problem. In the DNIP, there is a completely characterized flow network (i.e. configuration and nominal flow transmitted between any two nodes are known) for which there is a cost associated with the complete and effective interdiction of each link and also, a specified interdiction budget. The DNIP is optimally solved by identifying the network links that should be interdicted. Hence the flow between two specified nodes in a network—usually called source and sink—is minimized and, the interdiction cost is within the specified budget.

Although this original perspective of the DNIP was introduced more than four decades ago [1,2] it has become relevant in security related areas [3,4] as a means to address and prevent potential pernicious events caused by the existence of intentional attacks with the involvement of "a malevolent intelligence directed towards maximum social disruption" [5]. To address these new challenges, different implementations and variations of the traditional DNIP have been developed in recent years, among the most relevant: military operations [6], nuclear smuggling interdiction [7], border control [8], infection control in hospitals [9] and supply chain networks [10]. Also, the DNIP shares similarities with the optimization techniques used in the area of network survivability analysis [11–13].

These new perspectives on the DNIP have significantly contributed to the state-of-the-art in the area. However, they are still "traditional" in the sense that they have focused on optimizing a single function-of-merit (FOM) of the network design (usually, cost or source–sink flow) subject to known constraints on resources and/or network performance requirements.

Unfortunately, the traditional perspective of optimizing a single FOM does not allow the decision maker (DM) to concurrently contemplate different considerations (for example, to minimize network flow alongside the minimization of interdiction cost and the optimization of a function of the duration of interdiction) and on the impact these considerations have among each other. That is, when considering developing strategies for network interdiction, single-objective approaches cannot address diverse needs related to the possibility of having multiple competing objectives and multiple prospective solutions that may change based on the preference of the decision maker (DM). This paper intends to address these needs.

To address issues of competing optimization needs, multi-objective optimization (MO) has been proposed as an approach to solve the problem of finding solutions for mathematical models that have multiple objective functions with multiple optimization criteria. Unlike optimization models with a single objective function, where a solution may satisfy the optimization criteria

* Corresponding author.
E-mail addresses: jmarquez@stevens.edu, Jose.Ramirez-Marquez@stevens.edu (J. Emmanuel Ramirez-Marquez).

0951-8320/$ - see front matter
doi:10.1016/j.ress.2010.03.008

(i.e. become the optimal solution), the multi-objective case is concerned with obtaining solutions that best represent the conflicting nature of the different optimization criteria. That is, the interest is on finding a set of solutions (usually referred to as the Pareto-optimal set and each of its elements as a Pareto optimal solution) that describe the interaction of the different criteria or how the improvement of a single objective function value impacts the value of the other objectives.

Since to the best of our knowledge there is a lack of approaches to address a multi-objective view of the DNIP, this paper presents a new evolutionary algorithm that can be used to solve the MO-DNIP in general two-terminal directed and undirected networks. The optimization model considered for MO-DNIP takes into account different FOM such as cost, network flow and a function of interdiction duration. The algorithm to solve the MO version for DNIP offers simple and efficient analyses based on an evolutionary optimization approach known as the Probabilistic Solution Discovery Algorithm (PSDA).

When considering a single-objective perspective, PSDA is an evolutionary algorithm that, taking into account the analysis of initial solutions and in a probabilistic manner, iteratively explores regions of an optimization problem solution space with the intent of identifying an optimal solution. PSDA has been proven to yield high quality solutions for optimization problems as diverse as network interdiction [14], reliability allocation [15], container inspection [16] and ad-hoc wireless networking [17] to name a few.

For the MO-DNIP, this paper presents a new evolutionary algorithm called MO-PSDA that generates potential solutions (i.e. network interdiction strategies) based on an initial specified probability distribution and, for each of these solutions, their associated objective function values (i.e. cost, network flow and average flow during the interdiction period) are obtained. These solutions are then analyzed for Pareto optimality based on a comparison of their objective function values. Once this step is performed, the initial probability distribution—to generate potential interdiction strategies—is updated as a function of the current Pareto optimal solutions. The cycle is restarted until this distribution converges to a constant set of optimal solutions or a stopping criterion is enforced.

The purpose of developing MO-PSDA for the DNIP is two-fold.

First, it recognizes that when presented with an interdiction problem, the DM is sometimes faced not with a single objective to satisfy, but with multiple conflicting needs related to resources and interdiction performance for which different alternatives must be taken into account. That is, the DM is interested in selecting a solution in accordance with his/her preferences and needs. Even in cases where the DM is interested to know how to use all the resources to interdict the flow, the DM may benefit from understanding the trade-off between a strategy with higher interdicted flow and higher cost and a lower cost alternative that sacrifices flow interdiction.

Currently, to address this challenge, the DM must solve several problems using a SO approach by varying a group of constraints. Hence, for example, Royset and Wood [18] have analyzed a bi-objective DNIP that considers cost and network flow as the optimization criteria. Their approach is based on an algorithm that identifies the Pareto optimal set through a sequence of single-objective problems solved using Lagrangian relaxation and a specialized branch-and-bound algorithm.

Second, based on the needs of the DM, this manuscript provides a simple and powerful optimization approach that immediately allows characterizing the solutions—also known as the Pareto front for MO-DNIP—without the requirement to solve multiple single-objective optimization problems.

In summary, this manuscript contributes to the state-of-the-art in DNIP by providing a MO perspective to the DNIP, including an approach to quantify the duration of an interdiction strategy (to the best of the authors knowledge, traditional DNIP has not accounted for this FOM; a potentially serious drawback since in many NI applications the interdiction cannot be expected to be perpetual) and as previously discussed the algorithm developed for the solution of the MO-DNIP does not resort to potentially infinite recursive solutions of SO-DNIP.

The remainder of the paper is organized as follows. Section 2 presents a literature review of the DNIP solution approaches along with relevant MO literature. In Section 3, MO-DNIP is developed and the PSDA for its solution is proposed. Section 4 presents different test cases that show the accuracy and repeatability of the approach. Finally, Section 5 presents conclusions.

Notation

N	set of nodes
A	set of arcs
$l=\lvert A\rvert$	number of links
x_w	binary decision variable representing if the link w is interdicted ($x_w=0$) or not ($x_w=1$) in the network.
$\mathbf{x}$	interdiction strategy vector $\mathbf{x}=(x_1,x_2,\ldots,x_l)$
u	number of loops in the algorithm
$\mathbf{x}_u^h$	hth potential interdiction strategy
$\boldsymbol{\gamma}_u$	vector of probabilities, $\boldsymbol{\gamma}_u=(\gamma_{1u},\gamma_{2u},\ldots,\gamma_{lu})$
γ_{wu}	probability that link w is interdicted, $\gamma_{wu}=P(x_w=0)$
c_w	cost for interdicting link w
k_w	nominal flow for link w
$kmax_w$	maximum flow for link w
t_w	time to restore interdiction to nominal flow for link w
$C(\mathbf{x}_u^h)$	interdiction cost for the hth potential interdiction strategy
$F(\mathbf{x}_u^h)$	network $s-t$ maximum flow for the hth potential interdiction strategy
$R(\mathbf{x}_u^h\vert T_0)$	average flow during the mission time for the hth potential interdiction strategy during mission time T_0
S	size of a subset of solutions
H	optimal Pareto set of solutions
$\vee$, $\wedge$	or, and operators

Acronyms

DNIP	deterministic network interdiction problem
MC	Monte Carlo
GA	genetic algorithm
DM	decision maker
PSDA	Probabilistic Solution Discovery Algorithm
MO	multi-objective
MOEA	Multi-objective Evolutionary Algorithm
FOM	figures-of-merit
NI	network interdiction

Assumptions

1. link interdiction cost, nominal flow and restoration time, are known for every link;
2. $s-t$ network configuration is fixed and known;
3. no flow can be transmitted along any interdicted link;
4. link interdiction is always successful.

2. Literature review

2.1. Deterministic network interdiction research

As discussed NI has been relevant since the early 1960s when the DNIP was proposed. The McMasters and Mustin [6] solution