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On vibrational behavior of pulse detonation engine tubes

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A B S T R A C T

This paper presents a set of analytic solutions for the transient elastodynamic response of orthotropic cylindrical tubes to sequential moving pressures with specific profiles. The general form of the presented formulations and the solution methods are applicable to a number of theoretical and practical problems. However, the final solutions are tailored for sequential gaseous detonations, with direct application in the stress analysis of pulse detonation engines (PDE). The PDE generates trust by high cycling of gaseous detonations and is regarded as a promising candidate for providing very efficient propulsion systems for aviation and electric power generation. The presented analytic solutions are validated with the available experimental data and complementary finite element simulations. Representative analyses are carried out for an experimental detonation tube subjected to different boundary conditions and loading sequences. It is shown that the proper (or improper) combinations of the relevant parameters can result in substantial attenuation (or amplification) of the tube vibration. It is also demonstrated that the realistic analysis of the overall vibrational behavior, which can be highly affected by the specifications of the loadings and boundary conditions, requires the simulation of hundreds of sequential detonations.

1. Introduction

Formulation of the transient elastodynamic response of cylindrical tubes under sequential moving pressures has direct application in the vibrational analysis of pulse detonation engines (PDE) [1]. The PDE is currently regarded as a promising candidate for providing very efficient propulsion systems for aviation and also electric power generation [2–4] (see Fig. 1).

Detonations are combustion events in which the speed of the combustion wave front is supersonic. A detonation explosion is more severe than a deflagration explosion (with subsonic wave front) since the pressure waves are much stronger. Because of the dynamic effects of this high-speed moving load, the displacements in the detonation tube wall are *oscillatory*. Accordingly, various types of structural response and damage can occur in the tube wall [5], e.g., elastodynamic response and high-cycle fatigue at relatively low pressures [6], elasto-plastic deformation and dynamic tearing at medium pressures [7–11], and severe plastic expansion with dynamic fragmentation at high pressures [12,13].

In practice, PDEs generate quasi steady thrust by rapid cycling of relatively *low-pressure* gaseous detonations. Thus, a main issue in the stress analysis of the PDE tubes is the interactions between the *elastic stress waves* caused by sequential detonations. There have been several studies on the analysis of the vibrational behavior and critical speeds (which amplify dynamic stresses) of cylindrical tubes under moving pressures [14–24]. The first comprehensive theories were developed by Tang [14] and Reismann [15]. The model presented by Tang included the effects of transverse shear and rotary inertia and presented *steady state* solutions for an *infinite* tube. Simkins [16] extended the analysis to thick-wall cylinders to study the behavior of gun tubes. The experimental, analytical, and numerical studies carried out by Beltman and Shepherd [17] showed that the true characteristics of the response for *finite* tubes can only be revealed through *transient* models. They made two major simplifications to the governing equations of the Tangs model and also dropped the effects of transverse shear and rotary inertia to obtain a tractable transient formulation. Mirzaei et al. [18] modified the original Tangs formulation and developed a new *transient* analytic model for finite tubes, in which all the essential terms in the governing equation were preserved. However, their original solution was only valid during a time period of L/V (the time required for the detonation front to travel the entire length of tube). In practice, as the detonation loading leaves the tube, the free-vibration phase starts with the initial conditions inherited from the previous forced-vibration phase. The solution method for the above problem, which extended the solution beyond L/V and thereby enabled the simulation of the structural

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Nomenclature

E_x	Young's modulus in the x direction	N/m^2	t	time variable	s
E_θ	Young's modulus in the θ direction	N/m^2	$\tilde{t}$	general time variable	s
G	shear modulus	N/m^2	u	axial displacement	m
H	step function		V	load speed	m/s
h	tube thickness	m	V_{Dx}	dilatational wave speed in x direction	m/s
L	tube length	m	V_{Sx}	shear wave speed in x direction	m/s
m	detonation index		w	total radial deflection	m
M_{xx}	bending moment resultant	N	w_b	radial deflection due to bending	m
n	mode index		w_s	radial deflection due to shear	m
N_{xx}	stress resultant in the x direction	N/m	x	axial coordinate	m
$N_{\theta\theta}$	stress resultant in the θ direction	N/m	β	dimensionless tube thickness parameter	
P_1	pre-detonation pressure	Pa	κ	shear correction factor	
P_2	maximum-detonation pressure	Pa	ν	Poisson's ratio	
P_3	post-shock pressure	Pa	ρ	density	kg/m^3
P_{ext}	external pressure	Pa	Λ_j	dimensionless excitation parameters ($j = 1, 2, 3$)	
P_{cj}	Chapman–Jouguet pressure	Pa	ψ	rotation	
Q_x	shear stress resultant	N/m	Φ	dynamic amplification factor	
R	tube mean radius	m	Δ	time interval between detonations	s
T	exponential decay factor	s			

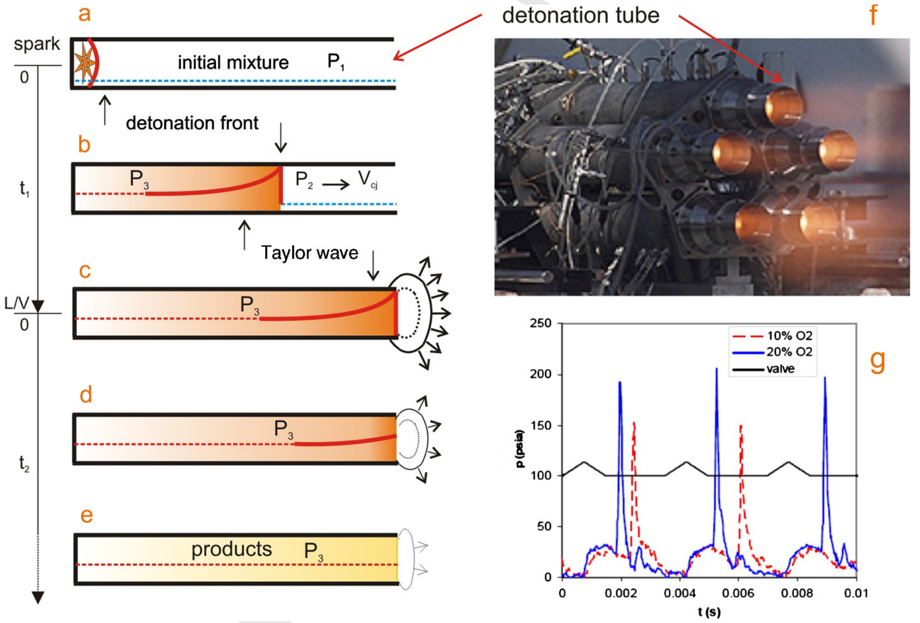

Fig. 1. (a)–(e) A PDE cycle, showing the detonation initiation at the thrust wall plus the propagation of the Taylor wave to the open end of the tube at the speed of V_{cj}. (f) A five-tube PDE test rig [4]. (g) A sequence of pressure traces in an experimental PDE running at 300 Hz [3].

waves reflected at both ends of the tube, was reported in Ref. [19]. The predictions of this model were quite comparable to the experimental results for a *stainless steel* detonation tube. Nevertheless, the derived solutions were based on a simplifying assumption that the free-vibration phase starts as soon as the *detonation front* reaches the end of the tube. In reality, when the detonation front leaves the tube, the loading effects of its trail still exist. Thus, an enhanced form of the solution, which considered the trail loading and accounted for various loading effects (pre-shock, post-shock, and external pressures), along with the analytic solution for transverse shear strain were developed [20]. This solution showed the occurrence of resonance in transverse shear stress of an *aluminum* detonation tube near the second critical speed. This resonance was several times stronger than the finite element (FE) results reported by Chao and Shepherd [21], who used a very fine mesh and very small time steps to be able to capture the shear stress resonance.

Recently, the analytic formulation of Ref. [20] was further modified (by definition of two time domains for progression of the solution in time) and extended for *sequential* detonation loadings [24]. The results of this new solution, along with complementary FE simulations, clearly showed that the vibrational behavior can be highly affected by the specifications of sequential loading and high dynamic amplification factors can exist even at noncritical speeds.

In the current study, the accuracy of the transient analytic model reported in Ref. [24] has been enhanced through a modified loading function and the formulation has been extended to handle *orthotropic* material behavior. The solutions are compared with the available experimental data and complementary finite element simulations. In continuation, the influences of the loading

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