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Reliability estimates for flawed mortar projectile bodies

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ABSTRACT

The Army routinely screens mortar projectiles for defects in safety-critical parts. In 2003, several lots of mortar projectiles had a relatively high defect rate, 0.24%. Before releasing the projectiles, the Army reevaluated the chance of a safety-critical failure. Limit state functions and Monte Carlo simulations were used to estimate reliability. Measured distributions of wall thickness, defect rate, material strength, and applied loads were used with calculated stresses to estimate the probability of failure. The results predicted less than one failure in one million firings. As of 2008, the mortar projectiles have been used without any safety-critical incident.

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1. Introduction

The Army routinely screens mortar bodies for manufacturing defects in critical areas. In 2003, as part of the Army's normal inspection procedure, a small percentage of defects were found in 60-mm mortar bodies, Fig. 1. The drawing specifies a wall thickness between 0.34 and 0.42 cm. The minimum wall thickness in non-conforming shells was 0.24 cm. In response to the finding, the Army screened the suspect mortar bodies at 100% rate by manual gaging the wall thickness. Structural and reliability analyses were completed to:

1 Estimate probability of yielding
2 Estimate the probability of a mortar projectile failure in a gun tube, a safety-critical event
3 Determine if the minimum wall thickness on the drawing was adequate

The probability of a failure in a gun tube was estimated to be less than 1E−8 and acceptable. A number of improvements in the manufacturing and inspection process were also made. This paper describes the structural and reliability study on the 60-mm mortar bodies.

2. Background

2.1. Limit state functions

Limit state functions provide a way to predict reliability as a function of physical equations and random variables. Generally, limit state functions take the form $g(X1,\ X2,) < \text{constant}$. The variations in the limit state functions provide a means to quantify the probability of failure.

As examples, Heitzer and Staat [1] tied a limit state function to a finite element analysis for in-elastic structural analysis. NASA

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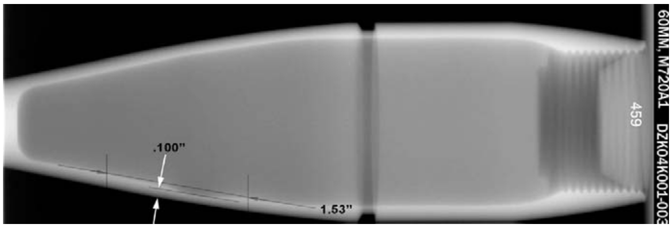

Fig. 1. Screened shell body with anomaly, 60 mm mortar.

scientists are using limit state functions for optimal wing design by combining aerodynamics, computational fluid mechanics, and finite element analysis [2–5]. Shah and Korovaichuk [6] used limit state functions to evaluate fasteners for space structures. Moglia et al. [7] evaluated the probability of a piping system failure using limit state functions. The Army used limit state functions to estimate the likelihood of tolerance stack-up failures in fuzes [8,9].

3. Statistical method

3.1. General approach

In this study, limit state functions were used to compare calculated stresses to material strengths. Reliability predictions were based on limit state functions, finite element results, statistical data, and Monte Carlo simulations. Several limit state functions were considered:

$$G1 = \text{strength-stress\_function} \tag{1}$$

$$G2 = \text{elongation-stress\_function} \tag{2}$$

$G1$ is the probability that stress exceeds material strength. Yield strength and ultimate tensile strength were evaluated separately. $G2$ is the probability that strain exceeds material elongation. The probability of failure is the probability that either $G1$ or $G2$ is less than zero.

Two commercially available software packages were used for analysis. DistributionProbe [10] was used determine the best statistical distribution to represent a list of strength and elongation values. The software package included 15 statistical distributions: beta, double exponential, exponential, gamma, Gumbel, Logistic, lognormal, Maxwell, normal, Pareto, Rayleigh, Type I smallest, Type II largest, uniform, and three-parameter Weibull. Three goodness-of-fit tests are available: Anderson–Darling Test, Cramer–von Mises test, and Kolmogorov–Smimov (K–S). The K–S test was based on the largest vertical distance between the empirical distribution and the probability distribution. The user inserted data points and chose a goodness-of-fit criterion. For this study, the K–S test was chosen. DistributionProbe checked the fit of the empirical data against the 15 probability distributions, ranked the distributions based on goodness-of-fit, and provided the statistical parameters for each distribution. The best distribution to represent the data was than used in the Unipass [11] software package to determine probability of failure.

The Unipass [11] package was used to predict the probability that the limit state functions were less than zero, failure. Unipass includes three methods for predicting probability of failure: 1st-order method, 2nd-order method, and Monte Carlo simulation. For this analysis, 1E6 Monte Carlo simulations were used. The 1st- and 2nd-order methods require fewer simulations to predict probability. Unipass input included random variables and limit state functions. Statistical distributions from the DistributionProbe package were used to model yield strength, ultimate tensile strength, and material elongation. A uniform distribution was used to account for geometry variations. The variation in pressure loads in the gun tube was obtained from experiments and known to be close to normal distributions.

For completeness, other modes of structural failure were ruled unlikely. Structures can fail one of three ways: yielding, buckling, or unstable crack growth [12]. The limit state functions G1 and G2 provide failure probability associated with yield failure. Finite element analysis was used to estimate the critical buckling load. The critical buckling force exceeded the 3-sigma compression load by a factor of 7 making buckling unlikely. Finite element analysis of the flawed mortar showed the stress at the flaw edges to be in compression. Since cracks do not grow when the crack tip stress is compressive, this failure mode was also ruled unlikely.

3.2. Strength and elongation functions

The mortar bodies are made of HF-1 steel. HF-1 steel was developed for the Army based on fragmentation requirements [13]. The mortar drawing called for a minimum yield strength of 553 MPa at 2% offset and a minimum elongation of 7%. There is no criterion for ultimate tensile strength for this particular mortar shell.

Limit state functions $G1$ and $G2$ used the material strengths and elongations. This empirical data were gathered from a well-established inspection method. The Army retains inspection reports for each heat treat lot. No field failures have been reported for lots that pass its inspection criteria.

The standard inspection procedure is as follows. For each heat treat lot, material hardness is tested at two locations in mortar bodies. (Hardness is an inexpensive, non-destructive test that correlates loosely with material strength). The projectiles with the highest and lowest hardness were chosen for destructive strength and elongation tests. For each projectile body, two tensile specimens were taken from the forward region and two specimens were taken from the rear taper. Roughly half the data are shown in Table 1. Averages and standard deviations differed slightly between locations and hardness groups. All yield strengths met or exceeded drawing requirements. The minimum elongation from tests, 9%, also exceeded the material requirements stated on the drawings.

For this study, data from 12 lots of heat-treated HF-1 steel was used to determine a statistical distribution. Lots were provided by others, not chosen based on statistical considerations. DistributionProbe [10] was used to determine the statistical distribution for the empirical data points. For the yield strength data, Type I largest Gumble provided the best fit for the three goodness-of-fit tests in DistributionProbe, Fig. 2. The correlation for Gumble using the Kolmogorov–Smirnov (K–S) goodness-of-fit test was 97%. Comparing, the goodness-of-fit for a Weibull and lognormal distribution were 62% and 28%, respectively. When the high-hardness and low-hardness data were evaluated separately, results were similar to the entire population.

Data points from the same 12 lots were also used to find a statistical distribution for elongation data. Using the K–S goodness-of-fit criteria, the best fit was to a Rayleigh distribution at 54%. The elongation distribution is shown in Fig. 3.

The ultimate tensile strength was difficult to determine for the 60-mm mortar shells. It was not tested, not specified on the drawing, not included in the material specification, and not given in the usual references for material properties [14]. The Army metallurgist at Picatinny Arsenal provided measured ultimate tensile strength data came from 11 tests from another project with HF-1 steel. The best curve fit was with a double-exponential distribution with a 93% goodness-of-fit. The average value was 1108-MPa and the standard deviation was 19.4-MPa. The ultimate