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Mathematical modeling and control of a tilt-rotor aircraft

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ABSTRACT

This paper presents a novel model of large-size tilt-rotor aircraft, which can operate as a helicopter as well as being capable of transition to fixed-wing flight. Aerodynamics of the dynamic large-size tilt-rotors based on blade element method is analyzed during mode transition. For the large-size aircraft with turboshaft engines, the blade pitch angles of the rotors are regulated to vary according to the desired level of thrust, and the following expressions are formulated explicitly: rotor thrust and blade pitch angle, drag torque and blade pitch angle. A finite-time convergent observer based on Lyapunov function is developed to reconstruct the unknown variables and uncertainties during mode transitions. The merits of this design include the modeling of dynamic large-size tilt-rotor, ease of the uncertainties estimation during the tilting and the widely applications. Moreover, a switched logic controller based on the finite-time convergent observer is proposed to drive the aircraft to implement the mode transition with invariant flying height.

1. Introduction

Helicopters and fixed-wing airplanes have their advantages and shortcomings. Helicopters can take off and land vertically, but they cannot fly forward in high speed, and their payloads are very limited comparing to fixed wing plane with the same gross weight [1–4]. On the other hand, conventional fixed-wing aircrafts can fly forward in high speed and large payloads. However, they cannot take off and land vertically, and the appropriate runways are required.

There are several ways to perform the vertical takeoff and landing (VTOL) maneuver such as tilting rotor, tilting wing, thrust vectoring, tail sitter, tilting fuselage, flapping wing, multi-propeller multifunction, etc. There are some types of thrust vectoring aircrafts, such as manned aircrafts AV-8B Harrier [5] and F-35 [6]. These aircrafts are designed for actual combat. The reason why these aircrafts can perform vertical takeoff and landing is due to their jet engines with tilt jettubes. Although they are powerful, the jet gas is very hot and harmful, and they can easily destroy the ground environment or inflict injuries to people nearby. These aircrafts are not suitable for many civil and rescue operations. Moreover, such VTOL aircraft with jet engines is less efficient in hover than a conventional helicopter or a tilt-rotor aircraft of the same gross weight [7].

One of the classical tilt-rotor aircrafts is V-22 [8]. Similarly the Bell Eagle Eye UAV [9] which is also based on tilt-rotor technology has a large success in the civil and military domains. However, this design brings its own problems, since the degradation in stability is usually observed in high speed forward flight (airplane mode) [10]. Moreover, the involved equations of motion are highly coupled and nonlinear. The fundamental characteristics of V-22, which includes a very high roll moment of inertia, result in a tendency to pilot-induced oscillation. Extreme attention must be paid to preventing over control in the roll channel. Moreover, the roll channel is easily affected by pitch dynamics. V-22 is a large "U"-shaped structure with two very large masses at the ends (the engines). The result is the need for long and flexible runs of hydraulic, electrical, and mechanical lines that are more susceptible to tensile loads, bending loads, abrasions, and deformations than conventional helicopter designs. Because of the high disk-loading (129.63 kg/m^2), the downwash velocity of V-22 is about twice that of any conventional helicopter. And because of the side-by-side placement of the prop-rotors, there are two distinct downwash wakes that are transverse to the flight direction. This has several operational implications that bear on safety issues.

Tail-sitters, as the name implies, sit on their tail when not in flight [11–14]. They take off and land vertically, making them a member of the VTOL family of aircraft. Equipped with a powerful engine, tail-sitters can utilize a "prop-hanging" technique to hover in place. Additionally, they can make transition to a level flight mode and fly in a conventional fixed-wing mode, which is more energy efficient than the hover mode. These aircrafts have

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Nomenclature

m_e	weight of each engine
T_e	rotor thrust under rated power
ρ_e	thrust-to-weight ratio of engine
m	gross weight of aircraft
T_i	rotor thrust
C_{T_i}	rotor thrust coefficient
Ω_i	rotor speed
ρ	air density
σ	rotor solidity
p	blade number
R	rotor radius
ρ_b	blade span-chord ratio
A	area of rotor disk
g	gravity acceleration
C_{d0}	constant profile drag coefficient
κ	induced power factor
b	chord length of blade
PL	power loading of rotor
DL	disk loading of rotor
R_{2r}	rotor radius of aircraft with two rotors
R_ε	Reynolds number
M	Mach number
DL_{QT}	Desired disk loading of rotor
A_w	span-chord ratio of fixed wing
WL_w	desired wing loading of fixed wing
S_w	wing area of fixed wing
L_w	wing span of fixed wing
C_w	chord length of fixed wing
S_{front}	area of front fixed wing root
S_{fi}	area of free wing
Γ_g	right handed inertial frame
Γ_b	frame attached to the aircraft's fuselage
Γ_β	right handed inertial frame of the tiltrotor
ψ	yaw angle
θ	pitch angle
ϕ	roll angle
R_{bg}	transformation matrix for Γ_b to Γ_g
$R_{\beta b}$	transformation matrix for Γ_β to Γ_b
$V_{\beta X}$	air relative velocity on axis E_x^β
$V_{\beta Y}$	air relative velocity on axis E_y^β
$V_{\beta Z}$	air relative velocity on axis E_z^β
V_b	air relative velocity with respect to frame Γ_b
v_i	induced velocity of rotor disk
v_h	induced velocity in hover
r	length from rotor center to blade element
β_*	relative inflow angle at blade element
α_*	angle of attack of blade element
φ_*	blade pitch angle
φ_{7i}	pitch angle at $r/R = 0.7$
$\Delta\varphi_*$	linear torsion of blade
U	resultant velocity at disk
dL	lift per unit span on blade element
dD	drag per unit span on blade element
C_l	lift coefficient of blade
C_d	drag coefficient of blade
dr	incremental radial distance of blade
dT_i	incremental lift of rotor
dQ_i	incremental drag torque of rotor
Q_i	drag torque of rotor
C_{Qi}	drag torque coefficient of rotor
T	sum of rotor thrusts
$V_{\beta t}$	airflow generated by rotor tilting
V_{rt}	resultant velocity at the free wing
α_f	angle of attack for free wing
d_f	distance between rotor center to leading edge of free wing
L_i	$(i = 1, 2, 3, 4)$ lift force generated by free wing
D_i	$(i = 1, 2, 3, 4)$ lift force generated by free wing
ΔL_i	$(i = 1, 2, 3, 4)$ uncertain lift force for free wing
ΔD_i	$(i = 1, 2, 3, 4)$ uncertain drag force for free wing
S_{fi}	area of free wing
C_f	lift coefficient brought by angle of attack α_f to free wing
C_{Df0}	drag coefficient when angle of attack α_f is equal to zero
δ_i	$(i = 1, 2, 3, 4)$ flap bias angle of free wing
C_r	lift coefficient brought out by flap bias angle δ_i
e_f	value of the Oswald's Efficiency Factor for free wing
A_f	span-chord ratio of free wing
$\alpha_{\max}$	angle of attack with respect to maximal lift coefficient of free wing
L_i	$(i = 5, 6)$ lift force generated by fixed wing
D_i	$(i = 5, 6)$ lift force generated by fixed wing
ΔL_i	$(i = 5, 6)$ uncertain lift force for fixed wing
ΔD_i	$(i = 5, 6)$ uncertain drag force for fixed wing
α	angle of attack for fixed wing
S_{ri}	area of the left (right) fixed wing
$C_{w\alpha}$	lift coefficient brought by angle of attack α to fixed wing
C_{w0}	lift coefficient when angle of attack α is equal to zero
C_{Dw0}	drag coefficient when angle of attack α is equal to zero
δ_i	$(i = 5, 6)$ flap bias angle of fixed wing
$C_{w\delta}$	lift coefficient brought out by flap bias angle δ_i
e_w	value of the Oswald's Efficiency Factor for fixed wing
f_{rl}	yaw force generated by vertical tail
f_{rd}	drag force generated by vertical tail
G_β	gyroscopic moment for tilting rotors in frame Γ_β
G_b	gyroscopic moment for tilting rotors in frame Γ_b
τ_T	thrust vectoring moment in frame Γ_b
C_i	$(i = 1, 2, 3, 4)$ center of rotor
Q	sum of drag torques of rotors in frame Γ_β
Q_r	sum of drag torques of rotors in frame Γ_b
τ_β	tilting pitch moment in frame Γ_b
τ_δ	moment generated by lift of fixed wings in frame Γ_b
τ_f	moment generated by free wings in frame Γ_b
τ_{fd}	moment generated by drag of fixed wings in frame Γ_b
τ_{xd}	moment generated by drag of free wings in frame Γ_b
J_1	moment of inertia with respect to axis E_x^β
J_2	moment of inertia with respect to axis E_y^β
J_3	moment of inertia with respect to axis E_z^β
J_4	tilting moment of inertia

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