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Adaptation and optimization of the synchronization gains in the adaptive spacecraft attitude synchronization

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ABSTRACT

This work considers various optimization aspects for the attitude synchronization of spacecraft formation. Revisiting the synchronization controllers that address unknown parameters via adaptive techniques, this work proposes a modification to the synchronization weights employed to enforce state agreement amongst the networked spacecraft. By augmenting a standard adaptive controller that accounts for unknown parameters, with adaptation of the synchronization weights, one opts to improve spacecraft synchronization. The interconnection strengths of the network nodes, responsible for enforcing synchronization amongst spacecraft, are weighted dynamically in proportion to the disagreement between the spacecraft states. Additionally, when all spacecraft are assumed identical, differing only in their orientation and initial conditions, a modification of the standard adaptive parameter laws allows to reach parameter consensus via a dynamic penalization on the pairwise differences of parameter estimates. The proposed adaptive control architecture which allows for adaptation of both parameter uncertainties and synchronization penalty terms is demonstrated via numerical studies of a four-spacecraft network with limited connectivity. As an alternative to the dynamic adjustment of the synchronization gains, a scheme is proposed to optimally select these gains. By considering the sum of deviation-from-the-mean and rotational kinetic energy as appropriate metrics for synchronization, the numerical studies provide insights into the selection of optimal edge-dependent synchronization gains when the initial conditions are assumed available.

1. Introduction

An aspect of spacecraft formation objective is the attitude synchronization problem, and this has attracted considerable interest, see for example [1–10] and the references therein. Enabling controller architecture includes modifications to account for parametric uncertainties. A way to address parametric uncertainties and disturbances, as for example externally generated environmental torques, is to employ adaptive control methods. Beyond stabilizability and tracking, which can be addressed by robust or adaptive controllers, the attitude synchronization problem requires additional control elements. They come in the form of additive terms in the controllers and consist of terms that penalize the mismatch between the spacecraft states. For example, the penalty terms in each controller may include the pairwise difference of spacecraft angular velocities, weighted by appropriately chosen penalty gains.

Theoretical explorations on the fundamental methods of non-linear control for mechanical systems have been an extremely active area of research in the last two decades. Literature [11–13] proposes tools for the systematic control design for non-linear mechanical systems. Many works on general dynamical systems examined various aspects of control design, ranging from output feedback to robust design [14–18], and they helped provide the foundation for extending the synchronization problem to spacecraft formation.

In paper [19], directed and switching topology for attitude synchronization problem is considered. Two assumptions about the measurable information are discussed. One is the absolute rotation, the other is relative rotation. Another paper [20] considers adaptive consensus for multiple systems with time-varying delay and uncertain parameters. The switching topology is based on the connected undirected topology. However, both papers do not concern the adaptation and optimization of the synchronization gains.

Specific to spacecraft, an adaptive controller for attitude synchronization based on Lyapunov theory was established in [9]. It entailed control theory for delay-free and coupling time-delay topologies to achieve attitude synchronization of spacecraft formation. The control architecture introduced allowed for parameter uncertainties. The adaptive tracking of Lagrangian systems was

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considered in [21]. The work in [22] addressed the cooperative tracking problem in the presence of model uncertainties and time-varying delay, especially the development of an output feedback control law without explicitly requiring the information of angular velocity.

The authors in [23] designed a distributed robust controller to address the attitude tracking problem of multiple spacecraft. An adaptive sliding mode controller was proposed to deal with the inertia matrices uncertainties. The work in [24] proposed adaptive schemes for unknown parameters in system dynamics in the way of coordinating torques and control laws by position and velocity errors and Chopra et al. [25] considered the problem of bilateral teleoperation with unknown parameters and developed a passive coordination control to synchronize the states of master and slave robots.

Continuing with possible improvements of the gains used for spacecraft synchronization, is the time adjustment of these synchronization gains for networked systems with general non-linear dynamics. When the "disagreement" of spacecraft i with spacecraft j is "smaller" than the disagreement of spacecraft i with spacecraft k, then the gain of the first disagreement difference should be less than that of the second pairwise difference. This would allow for significant reductions in controller magnitudes and provide additional robustness due to uncertainties. This aspect is being considered here.

The contribution of this work is fourfold and is stated as follows:

1. It provides greater flexibility in the choice of the gains used in the synchronization signal and the controller torques by allowing each agent (spacecraft) to use different gains in each of the pairwise state differences used to dynamically enforce state agreement. This extends the work in [9] that used constant and uniform-with-respect to agents (i.e. node-independent) gains to the case of *edge-dependent synchronization gains*.
2. A scheme for optimally selecting these constant synchronization gains is summarized and which links the optimization of synchronization to control performance (tracking or regulation).
3. Third, it extends the earlier work [26–28], to include uncertainty in the inertia matrix and the external disturbance. An adaptive controller employed to address the parametric uncertainties is augmented with the *adaptive edge-dependent synchronization gains* to improve both the control and synchronization objectives.
4. Finally, for the case of identical spacecraft, the adaptive estimation scheme is modified to allow *consensus in the adaptive parameter estimates*.

We first formulate the problem in Section 2, and in Section 3 we present the results on basic optimization of constant edge-dependent synchronization gains. A method to select the constant synchronization gains by minimizing a measure of synchronization and a measure of controller performance is summarized in Section 3.1. The case of time adaptation of edge-dependent gains is considered in Section 4 and the stability of attitude synchronization is presented. When the spacecrafts are identical, thereby having the same unknown parameters, a modification presented in Section 4.1 provides consensus in the adaptive parameter estimates. Numerical studies are presented in Section 5 and conclusions follow in Section 6.

2. Problem formulation

2.1. Attitude dynamics

We consider a system of N networked rigid bodies (spacecraft) and using the Euler rotational equations of motion to describe the identical dynamics for each spacecraft, we obtain the following dynamical equation resolved in body frame

$$\mathbf{J}\dot{\boldsymbol{\omega}} - (\mathbf{J}\boldsymbol{\omega}) \times \boldsymbol{\omega} = \mathbf{u} + \mathbf{d}_{ext}, \tag{1}$$

where $\mathbf{J} \in \mathbb{R}^{3\times 3}$ is the total inertia matrix and $\boldsymbol{\omega} \in \mathbb{R}^3$ is the angular velocity vector [29]. The signals $\mathbf{u} \in \mathbb{R}^3$ and $\mathbf{d}_{ext} \in \mathbb{R}^3$ denote the control and external disturbance torques, respectively. The moment of inertia and external disturbance are assumed to be constant but unknown.

To bring the above in a form that is conducive to parametrization and also facilitate the synchronization and control design objectives, the orientation of spacecraft with respect to the inertial frame will be described by the Modified Rodrigues Parameters [30–32]. Consequently, the attitude vector $\mathbf{q} \in \mathbb{R}^3$ is $\mathbf{q}(\widehat{\mathbf{n}}, \theta) = \tan(\frac{\theta}{4})\widehat{\mathbf{n}}$, where $\widehat{\mathbf{n}}$ is the eigenaxis unit vector and $-2\pi < \theta < 2\pi$ is the eigenangle [31]. Therefore, the attitude vector $\mathbf{q}$ and the angular velocity $\boldsymbol{\omega}$ have the following relationship $\dot{\mathbf{q}} = Z(\mathbf{q})\boldsymbol{\omega}$, where

$$Z(\mathbf{q}) = \frac{1}{2}\Big(\frac{1}{2}(1 - \mathbf{q}^T\mathbf{q})\mathbf{I}_3 + \mathbf{q}\mathbf{q}^T + S(\mathbf{q})\Big),$$

and the skew-symmetric matrix $S(\mathbf{q})$ is defined as

$$S(\mathbf{q}) = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}.$$

The Euler equation (1) describes the rotational motion of each of the N networked spacecrafts, with angular velocity vectors indexed by $\boldsymbol{\omega}_i$, $i = 1, \ldots, N$. Using the above, the attitude spacecraft dynamics can be expressed through the Euler–Lagrange formulation [33]

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i = \boldsymbol{\tau}_i + \boldsymbol{\tau}_{ext,i}, \quad i = 1, \ldots, N, \tag{2}$$

and

$$\begin{aligned}
&\boldsymbol{\tau}_i = Z^{-T}(\mathbf{q}_i)\mathbf{u}_i, \quad \boldsymbol{\tau}_{ext,i} = Z^{-T}(\mathbf{q}_i)\mathbf{d}_{ext,i}, \\
&\mathbf{M}_i(\mathbf{q}_i) = Z^{-T}(\mathbf{q}_i)\mathbf{J}_i Z^{-1}(\mathbf{q}_i), \\
&\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = -Z^{-T}(\mathbf{q}_i)\mathbf{J}_i Z^{-1}(\mathbf{q}_i)\dot{Z}(\mathbf{q}_i)Z^{-1}(\mathbf{q}_i) \\
&\qquad\qquad - Z^{-T}(\mathbf{q}_i)S(\mathbf{J}_i\boldsymbol{\omega}_i)Z^{-1}(\mathbf{q}_i).
\end{aligned}$$

The above equation is linearly parametrizable as long as $\mathbf{d}_{ext}$ is constant. Thus, equation (2) enjoys some fundamental properties summarized below [34–37].

Property 1 *(P1). The inertia matrix $\mathbf{M}_i(\mathbf{q}_i)$ is lower and upper bounded, i.e. $\forall i = 1, \ldots, N$, one has $0 < \lambda_{min}\{\mathbf{M}_i(\mathbf{q}_i)\}\mathbf{I}_3 \le \mathbf{M}_i(\mathbf{q}_i) \le \lambda_{max}\{\mathbf{M}_i(\mathbf{q}_i)\}\mathbf{I}_3 < \infty$.*

Property 2 *(P2). The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric, that is for any vector $v \in \mathbb{R}^3$ one has $v^T\big(\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\big)v = 0$, $i = 1, \ldots, N$.*

Property 3 *(P3). The Coriolis term $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is bounded in the sense that $\forall \mathbf{q}_i, \dot{\mathbf{q}}_i$, $\exists k > 0$ such that $|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i| \le k|\dot{\mathbf{q}}_i|^2$, $\forall i = 1, \ldots, N$.*

Property 4 *(P4). With constant $\mathbf{d}_{ext}$ and inertial moments, the systems in (2) are linearly parameterizable [9], i.e. each system admits the expansion*

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