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Distributed cooperative guidance of multiple anti-ship missiles with arbitrary impact angle constraint

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ABSTRACT

The problem of distributed cooperative guidance with arbitrary impact angle constraint is addressed. A new distributed cooperative guidance law, which consists of a local control term to achieve zero miss distance and the desired impact angle, and a cooperative control term to achieve the consensus of impact time, is proposed. A sufficient condition in terms of the properties of the arbitrary time-varying sensing/communication topologies is established to achieve the consensus of impact time. Furthermore, the sufficient condition is extended to the case of the specific topology of leader-followers. Compared with the existing results, the proposed sufficient condition is less restrictive, making the proposed guidance law be capable of real implementation. Numerical simulations are presented to illustrate the effectiveness of the proposed guidance law.

1. Introduction

When using missiles to attack a warship, the impact angle and impact time are important constraints to increase the effectiveness of warheads. Impact angle constraint can be used to specify the direction of attack so that the missile can attack the most vulnerable parts of the warship. And it can also be used to facilitate a salvo attack by missiles coming from different, specified directions. Whereas, impact time constraint is particularly important for salvo attack [5,6], which is regarded as a cost-effective and efficient countermeasure to survive the threats of the defensive systems of warship.

The issue of impact angle control guidance (IACG) has been addressed in an amount of works, from the biased proportional navigation guidance based IACG laws [8,9,13,21,31] to the optimal control theory based IACG laws [12,22,23], from the model predictive static programming technique based IACG laws [1,16] to the sliding mode control theory based IACG laws [4,10,11,14,20,28,32], too numerous to mention one by one.

While the problem of impact angle control has received a large amount of attention, in contrast there has been relatively rare studies on the problem of impact time control (or salvo attack, or simultaneous attack). Conceptually, a simultaneous attack of multiple missiles against a single common target can be achieved by two ways. One way is the individual homing, in which a common impact time is commanded to all missiles in priori, and thereafter each missile tries to attack the target on time independently [2,3,5,15,25,29,30]. The way of individual homing is an open-loop control one, and is not robust to external disturbance, as pointed out in [24]. This motivates the study of another way to achieve simultaneous attack, that is, cooperative homing, in which the missiles rely on the sensing/communication network to exchange information and synchronize the impact time [6,24,26,27]. Zhao et al. [24] presented a hierarchical cooperative guidance architecture, and derived both centralized and distributed coordination algorithms to achieve simultaneous attack of multiple missiles based on the impact time control guidance (ITCG) law in [5]. By introducing the concept of time-to-go variance of multiple missiles, Jeon et al. [6] proposed the so-called cooperative proportional navigation guidance (CPNG) law, which can achieve a simultaneous attack by decreasing the time-to-go variance cooperatively till the intercept. Using the range-to-go and closing velocity as the coordination variables, Sun et al. investigated the consensus problem of multiple missiles with time-delays and switching leader-followers topologies [26]. Feedback linearization technique was used to linearize the nonlinear engagement dynamics, which is a key to facilitate the design of distributed cooperative guidance law. Wang et al. [27] also presented an integrated guidance and control strategy for cooperative attack of multiple missiles, in which a cooperative strategy expressed by the desired target look angle command was proposed and a robust controller was designed using dynamic surface control theory to track the desired target look angle command.

It should be pointed out that, all of the aforementioned results on the cooperative guidance [6,24,26,27] didn't take the im-

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pact angle constraint into consideration. When the impact angle constraint (especially large impact angle constraint) presents, the small lead/heading angle assumption used in [6,24] (which greatly simplifies the guidance law synthesis) is invalid. Also, the feedback linearization technique used in [22] is hard to be applied when impact angle constraint presents. Besides, the cooperative strategy presented in [27] is hard to be extended to include the impact angle constraint as well. It should also be pointed out that, all of the aforementioned results on the cooperative homing are derived with rather restrictive sensing/communication topological requirements (i.e., fully-interconnected [6,27], strongly connected and balanced [24], or leader-followers [26]). However, it is well established that missiles have to fly in a hostile environment, where serious electromagnetic interference (EMI) and interception of anti-air missile can be expected. Hence, the exchange of information among the missiles is limited and occurs only locally and intermittently. And the topology of sensing/communication network is time-varying and cannot be predicted or prescribed or known a priori. In such a context, it is of paramount importance to relax the topological requirement on sensing/communication network, so that the cooperative guidance law can be applied in the practical environment of battlefield.

As a natural progression of our recent work in [30] and [31], in this paper, we focus upon the design of distributed cooperative guidance law with arbitrary impact angle constraint. Specifically, a special architecture of cooperative guidance law is first proposed, based on which the dynamics of networked system is derived without using the small angle assumption or the feedback linearization technique but by neatly using the special structure of the guidance law. A cooperative control term is then constructed to achieve the consensus of impact time, and the convergence of the closed-loop networked system is analyzed. Compared with the listed literature, the main contributions of this paper are as follows: (i) the proposed guidance law not only ensures the consensus of impact time cooperatively but also guarantees the arbitrary desired impact angle; (ii) the sensing/communication topological requirements is relaxed so that the proposed guidance law is more feasible in practice; (iii) the convergence of the closed-loop networked system is proved under mild assumptions.

The remainder of this paper is organized as follows. The problem of cooperative guidance with impact angle constraint under time-varying unpredictable sensing/communication network is formulated in Section 2, and the necessary preliminaries are introduced as well. In Section 3, the distributed cooperative guidance law with impact angle constraint is designed. The conditions on sensing/communication network to achieve consensus of impact time are established, and the convergence of the closed-loop networked system is proved under mild assumptions. Numerical simulation results are presented in Section 4. Section 5 offers some conclusions.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider n anti-ship missiles attack a stationary target as shown in Fig. 1, where the missile M_i, $(i = 1, \cdots, n)$, has constant speed and the target T is stationary. Note that, the missile considered in this paper is aerodynamic controlled missile. That is, only the direction of speed can be controlled by the aerodynamic force, while the axial speed of the missile is uncontrollable. Hence, the guidance command a_i is perpendicular to V_i. And the heading angle can be calculated from

$$\dot{\theta}_i = a_i / V_i \tag{1}$$

The rate of line of sight (LOS) angle can be obtained by

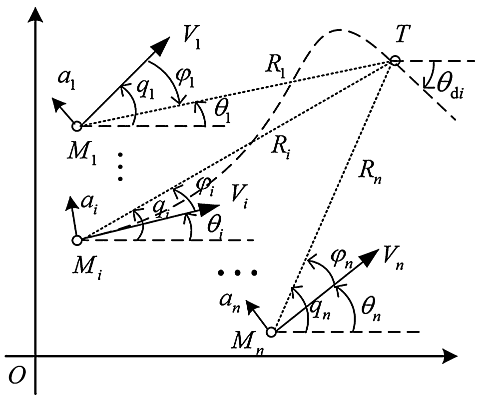

Fig. 1. Engagement geometry.

$$\dot{q}_i = V_i \sin\varphi_i / R_i \tag{2}$$

where R_i is the range-to-go, $\varphi_i = q_i - \theta_i$ is the lead angle.

It follows that, the engagement dynamics of each missile can be expressed in terms of R_i and φ_i as [6,26]

$$\dot{R}_i = -V_i \cos\varphi_i \tag{3a}$$

$$\dot{\varphi}_i = V_i \sin\varphi_i / R_i - a_i / V_i \tag{3b}$$

Remark 1. Note that, similar to Refs. [5,6,26], in the engagement dynamics Eq. (3), the target is modeled as being stationary since the maneuverability and the speed of surface ships are not comparable with those of anti-ship missiles of high-subsonic or supersonic speed. And it is assumed that the missile speed V_i is constant and the autopilot lag is negligible since the missile's time constant is small compared to the target's time constant [5].

The impact angle θ_{di} is defined as the heading angle at the final time of engagement, as shown in Fig. 1. The impact time $t_{\text{imp}i}$ can be given as

$$t_{\text{imp}i} = t_{\text{elap}i} + t_{\text{go}i} \tag{4}$$

where $t_{\text{elap}i}$ and $t_{\text{go}i}$ are the time elapsed and the time-to-go for the i-th missile, respectively. Note that, $t_{\text{go}i}$ cannot be measured by any onboard device, and its reasonable value can only be estimated by using some algorithm. It is assumed in this paper that the only information exchanged among missiles is the impact time and the exchange occurs only locally and intermittently.

To describe the maximum disagreement of impact time among the group of missiles, the following definitions are needed. Let $\Omega \triangleq \{1, \cdots, n\}$ be the set of indices on the missiles. At any instant of time t, three sets can be defined according to the value of impact time. That is, $\Omega_{\max}(t) \triangleq \{i \in \Omega : t_{\text{imp}i}(t) = t_{\text{imp}}^{\max}(t)\}$, $\Omega_{\text{mid}}(t) \triangleq \{i \in \Omega : t_{\text{imp}}^{\min}(t) < t_{\text{imp}i}(t) < t_{\text{imp}}^{\max}(t)\}$, $\Omega_{\min}(t) \triangleq \{i \in \Omega : t_{\text{imp}i}(t) = t_{\text{imp}}^{\min}(t)\}$, where $t_{\text{imp}}^{\max}(t) \triangleq \max_{j\in\Omega} t_{\text{imp}j}(t)$, $t_{\text{imp}}^{\min}(t) \triangleq \min_{j\in\Omega} t_{\text{imp}j}(t)$. Then, the maximum disagreement of impact time among the group of missiles can be given as

$$\delta_{\max}(t) = t_{\text{imp}i^*}(t) - t_{\text{imp}k^*}(t), \quad i^* \in \Omega_{\max}(t),\ k^* \in \Omega_{\min}(t) \tag{5}$$

With the above definitions, the problem considered in this paper can now be depicted as: To design a guidance command $a_i = a_i(t, R_i, V_i, \varphi_i, s_{i1}(t)t_{\text{imp}1}, \cdots, s_{ij}(t)t_{\text{imp}j}, \cdots, s_{in}(t)t_{\text{imp}n})$, using all the information available to the i-th missile (where $s_{ij}(t)$ are binary time functions, $s_{ii} \equiv 1$; $s_{ij}(t) = 1$ if $t_{\text{imp}j}(t)$ is known to the i-th missile at time t, and $s_{ij}(t) = 0$ if otherwise), such that for some unspecified final time of engagement t_{f},