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Attitude stabilization of rigid spacecraft with minimal attitude coordinates and unknown time-varying delay ☆

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ABSTRACT

The delayed feedback stabilization of rigid spacecraft attitude dynamics in the presence of an unknown time-varying delay in the measurement is addressed. The attitude representation is parameterized using minimal attitude coordinates. The time-varying delay and its derivative are assumed to be bounded. By employing a linear state feedback controller via a Lyapunov–Krasovskii functional, a general delay-dependent stability condition is characterized for the closed-loop parameterized system in terms of a linear matrix inequality (LMI) whose solution gives the suitable controller gains. An estimate of the region of attraction of the controlled system is also obtained, inside which the asymptotic stability of parameterized system is guaranteed.

1. Introduction

Feedback stabilization of rigid body attitude dynamics is an important control problem, see e.g. [1–5], with a wide range of applications such as spacecraft attitude maneuvers [6,7], underwater vehicles [8], and robotic manipulators [9]. The attitude representation depends on the choice of attitude parameters used to represent the orientation of a rigid body relative to an inertial frame see, e.g. [10–12]. Several control laws have been developed for the control of rigid body attitude dynamics. In [13,14], geometric controllers are designed on SO(3), which is the set of special orthogonal matrices, in order to track attitude and angular velocity commands while guaranteeing almost global asymptotic stability. In [2] an optimal controller is used, based on minimal attitude coordinates, to minimize a quadratic cost function for a dynamical system. Tracking control of a rigid asymmetric spacecraft is addressed in [7] by using a Hamiltonian–Jacobi formulation.

On the other hand, in several practical applications, there is an unavoidable time delay within the control system due to delay in measurements or actuators. Controlled systems designed based on feedback schemes are usually robust to a small amount of time delay. However, if the time delay increases due to the failure of system components or external sources, then the effect of this large time delay on the undesirable behavior of the controlled attitude motion is notable and may lead the non-delayed-based controlled system to chatter or produce oscillatory motion [15,16]. However, to the authors' knowledge, there are few studies on delayed feedback control of attitude dynamics [15,17–22]. In [17] a nonlinear robust controller is implemented without angular velocity measurements in the presence of a constant time delay in the control signal. The closed-loop system is shown to be stable for a norm bounded nonlinear uncertainty in the attitude dynamics. In [15], a velocity free output-based controller for attitude regulation of a rigid spacecraft considering the effects of a known time delay in the system is investigated. Sufficient conditions for attitude stabilization of the spacecraft are also obtained based on previously established controllers for manipulators. However, the asymptotic stability of system is only guaranteed for a sufficiently small time delay, while only a small estimate of the region of attraction was obtained. This conservatism has been fairly addressed in [21] by employing a linear state feedback controller for the attitude motion with an unknown time delay, which has a known upper bound in the feedback path using a frequency domain approach. A complete type Lyapunov–Krasovskii functional is constructed to ensure the robust stability of the linear controller and an estimate of the region of attraction is obtained.

In this paper, the delayed feedback control of rigid spacecraft attitude dynamics is studied. Kinematic differential equations of the spacecraft are modeled using minimal attitude coordinates that

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can include well-known attitude parameters such as Euler angles, classical Rodriguez parameters (CRPs), modified Rodriguez parameters (MRPs), and exponential coordinates. We assume that there is an unknown time varying delay in the measurement (as opposed to actuator delay [18,19]) with known upper bounds for both time delay and its rate. Unlike in [21], the controller gain matrices for the linear delayed feedback control law are obtained using a Lyapunov–Krasovskii functional in terms of a linear matrix inequality (LMI) which guarantees local asymptotic stability of the parameterized system. To cope with the nonlinear term in the dynamical model, we assume that the nonlinearities satisfy a nonlinear growth condition. Furthermore, an estimate of the region of attraction of the system is also obtained. Finally, a set of simulations is performed for a given set of spacecraft parameters.

This paper is organized as follows: In Section 2, we present the attitude kinematics and dynamics model. Section 3 presents the delayed feedback controller design, while an estimate of the region of the attraction of the system is obtained in Section 4. Numerical simulation results are shown in Section 5, and Section 6 concludes the paper.

2. Spacecraft attitude dynamics

In this section, we introduce some preliminary concepts of the minimal attitude parameterization $\xi \in \mathbb{R}^3$, and then introduce the kinematic and kinetic differential equations of rigid spacecraft. Two coordinate frames in the three-dimensional Euclidean space are employed. $\mathcal{N}$ defines the inertial coordinate frame and $\mathcal{B}$ defines the body-fixed coordinate frame. The spacecraft is modeled as a rigid body. In addition, we suppose that there are three actuators acting along orthogonal axes in the frame $\mathcal{B}$. In general, the equations of motion obtained by using the minimal set of attitude coordinates $\xi \in \mathbb{R}^3$ can be expressed as

$$\dot{\xi}(t) = \frac{1}{\beta}(\mathfrak{G}(\xi(t)) + I_3)\omega(t), \tag{1a}$$

$$J\dot{\omega}(t) = -\omega(t)^{\times} J\omega(t) + u(t), \tag{1b}$$

where $\mathfrak{G} : \mathbb{R}^3 \to \mathbb{R}^{3\times3}$ is a nonlinear function of ξ, β is a constant scalar, which is chosen according to the choice of attitude coordinates, $\omega(t) \in \mathbb{R}^3$ represents the angular velocity of the system described in $\mathcal{B}$ relative to $\mathcal{N}$, J is the known 3×3 constant positive definite inertia matrix of spacecraft, $u(t) \in \mathbb{R}^3$ is the control torque input, $I_3 \in \mathbb{R}^{3\times3}$ is the identity matrix, and $(\cdot)^{\times}$ is defined as

$$\omega^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \tag{2}$$

Equation (1a) is the kinematic differential equation, which can be considered as Euler angles, CRPs, MRPs, or exponential coordinates. Equation (1a) is also analogous to Poisson's equation $\dot{C}(t) = -\omega(t)^{\times} C(t)$ where $C(t) \in \mathrm{SO}(3)$ is the direction cosine matrix that describes the attitude of spacecraft from $\mathcal{N}$ to $\mathcal{B}$ and SO(3) is the set of all direction cosine matrices. Equation (1b) represents the Euler's rotational equations of motion.

Let the state variable $x(t)$ be defined as $x(t) = [x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t)]^{\mathrm{T}} = [\xi^{\mathrm{T}}(t), (1/\beta)\omega^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^6$. Eq. (1) can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x(t)), \tag{3}$$

where

$$A = \begin{bmatrix} 0_{3\times3} & I_3 \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} \in \mathbb{R}^{6\times6}, \qquad B = \begin{bmatrix} 0_{3\times3} \\ (1/\beta)J^{-1} \end{bmatrix} \in \mathbb{R}^{6\times3},$$

$$f(x(t)) = \begin{bmatrix} f_1(x(t)) \\ f_2(x(t)) \end{bmatrix} = \begin{bmatrix} \mathfrak{G}(x_1(t))x_2(t) \\ -\beta J^{-1} x_2(t)^{\times} J x_2(t) \end{bmatrix} \in \mathbb{R}^6. \tag{4}$$

The equilibrium subspace of the attitude motion is obtained as the set $E = \{(x_1, x_2) |\ x_1 \in \mathbb{R}^3, x_2 = 0\}$. The origin of the parameterized system, i.e., $x_1 = x_2 = 0$ is considered as the desired equilibrium point of the system. We seek to design a controller for the attitude dynamics in the presence of time delay in the measurement such that the origin is asymptotically stable.

Any set of minimal attitude parameterizations contains at least one geometrical orientation where the attitude is singular, see e.g. [4,11,12]. For example, MRPs have a singularity after one complete revolution. A singularity-free representation can be obtained if shadow set switching, which is an alternate set obtained from the projection of the other Euler parameters set, is performed [4]. However, switching is not employed in this study due to additional difficulties (e.g. chattering) resulting from discontinuous control in the presence of time delay. In addition, the exponential coordinates, which is a local diffeomorphism (non-singular) at the identity [23,24] and is obtained from the exponential map contains an ambiguity when the spacecraft rotates by $\Phi = \pi$ rad [25], where Φ is the principal rotation angle with the corresponding principal rotation axis $\hat{e}$.

Regarding the nonlinear vector $f(x)$, we have the following lemma:

Lemma 1. *The nonlinear function $f(x)$ defined in Eq. (3) satisfies $f(0) = 0$ and can be rewritten as $f(x) = F(x)x$, where $F(x)$ is a smooth function and is obtained as*

$$F(x) = \begin{bmatrix} 0_{3\times3} & \mathfrak{G}(x_1) \\ 0_{3\times3} & -\beta J^{-1} x_2^{\times} J \end{bmatrix} \in \mathbb{R}^{6\times6}. \tag{5}$$

Denote the induced 2-norm (see, i.e., [26]) of $F(x)$ by $\|F(x)\|_2 = \gamma(x)$, where $\gamma(x)$ is a positive real-valued function. We consider the neighborhood $\mathcal{N} \subset \mathbb{R}^6$ of the origin such that $\mathcal{N} = \{x \in \mathbb{R}^6 : \|x\| \le k\}$, where $k > 0$ is a known constant and the vector norm $\|.\|$ represents the Euclidean norm $\|.\|_2$. Therefore $f(x)$ satisfies a bounded growth condition in $\mathbb{R}^6$ such that $\|f(x)\| \le \gamma(x)\|x\| \le \gamma(k)\|x\|$. In addition, there exists a positive constant γ_{max} such that $0 \le \gamma(k) \le \gamma_{max}$.

The proof of this lemma will be discussed in Section 4.

3. Delayed feedback controller design

The objective of controller design is to stabilize the parameterized spacecraft model of Eq. (3) such that all the angular velocities and attitude parameters go to zero as $t \to \infty$ in some region of the domain $\mathbb{R}^6$ that contains the origin in the presence of an unknown time-varying delay in the feedback path, i.e., $\lim_{t\to\infty}\{\|x(t)\|^2\} = 0$. Thus, we design a linear controller for the nonlinear model of Eq. (3) by using the Lyapunov–Krasovskii method in conjunction with a LMI such that the local asymptotic stability of the solution is fulfilled. We assume that there is an unknown continuous time-varying time delay function $\tau(t) \in \mathbb{R}$ in the feedback loop such that

$$0 \le \tau(t) \le \tau_{max}, \quad \dot{\tau}(t) \le d < 1, \quad \forall t \ge 0, \tag{6}$$

where τ_{max} and d are positive constant scalars. Thus, all the measured signals that go to the controller are driven by the time delay. In addition, there are no control torque constraints.

3.1. Preliminary results

According to the above discussion, the linear state feedback control torque can be chosen as

$$u(t) = J[\beta K_1 \xi(t - \tau(t)) + K_2 \omega(t - \tau(t))] = K x_{\tau}, \tag{7}$$

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