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Adjoint-based adaptive finite element method for the compressible Euler equations using finite calculus

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ABSTRACT

In this paper, an adjoint-based error estimation and mesh adaptation framework is developed for the compressible inviscid flows. The algorithm employs the Finite Calculus (FIC) scheme for the numerical solution of the flow and discrete adjoint equations in the context of the Galerkin finite element method (FEM) on triangular grids. The FIC scheme treats the instabilities normally generated in the numerical solution of the fluid equations through adding two stabilization terms, called streamline term and transverse term, to the original central-based discretized formulation. The non-linear system of equations resulting from the flow problem is solved implicitly using a damped Newton's method accompanied with the exact Jacobian matrix. A defect corrected scheme is implemented to iteratively solve the linear system of equations related to the adjoint problem benefiting from the transpose of the Jacobian matrix. At each iteration, the linear systems of equations resulting from the fluid and adjoint problems are solved using a preconditioned GMRES method. Having calculated the error of a specified output functional locally, an *h*-refinement methodology based on the element subdivision is performed to refine the candidate elements. The quality of the numerical results proves the capability of the presented approach for the adjoint-based error estimation and mesh adaptation problems in different flow regimes.

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1. Introduction

Adaptive mesh refinement (AMR) is one of the most efficient means for reducing the overall computational cost in the numerical solution of engineering fluid problems modeled by the compressible Euler and Navier–Stokes equations [1–4]. The basic idea behind AMR is the control of the mesh resolution by generating an appropriate fine mesh near the zones where the solution error is high and assigning a coarse mesh to the rest of the domain. These techniques are able to improve the accuracy of the flow solution around the high-error zones such as boundary layers, stagnation points and shock waves. This effect of AMR is more remarkable when the numerical solution of the fluid flow is to be employed in an optimization problem [5] where an appropriate evaluation of a practical output function such as the lift and drag coefficients becomes important.

The main components of any AMR technique are a reliable error estimator/indicator and a mesh refinement methodology. The error estimator/indicator introduces areas from the current mesh where refinement is needed whereas the enhancement of the current mesh in these areas through adding new elements is the task of the mesh refinement methodology. Based on the so-called feature-based methods, one can consider the flow gradients [4,6] or flow curvatures [3,7,8] as the error indicator for predicting the areas where the refinement is needed. In these areas, the fluid flow mostly has some complex features such as shock waves, boundary layers and stagnation points. Although this family of error indicators can predict the flow features accurately, it does not necessarily provide an efficient estimation of the practical engineering outputs (such as lift and drag) used widely during the design optimization process.

In order to resolve this drawback, another family of error estimators/indicators, called output-based methods, has been developed recently employing the sensitivities of a specified output functional with respect to the flow solution where these sensitivities are predicted through the implementation of the adjoint variables. The general idea of these methods is to estimate the global error of the output functional as an inner product of the flow residuals and the adjoint variables a posteriori. For this end, two sets of problems, namely the flow problem and the adjoint problem, need to be solved on the current coarse mesh. The numerical solution of the flow equations provides the flow variables, whereas the adjoint variables are evaluated through the numerical solution of the adjoint equations.

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Besides the application of output-based methods for estimating the global error, they can be considered as a local error indicator to find the zones where the functional error is more than a desirable tolerance. The application of the output-based error estimation and mesh adaptation methods using adjoint variables in the context of finite volume is studied by Pierce and Giles [9] for the Poisson equation as well as the nonlinear quasi-one-dimensional Euler equations. At the following, Pierce and Giles [10] investigated this approach for two-dimensional inviscid shocked problems. The extension of this method for compressible two-dimensional inviscid and viscous flows is delivered by Venditti and Darmofal [11,12]. Park [13] employed this approach for incompressible and compressible three-dimensional Euler problems whereas Nemec et al. [14] demonstrated this for complex geometries. In the context of the finite element discretization method, Becker and Rannacher [15] developed adjoint-based error estimation and mesh adaptation for viscous fluid flow, chemically reactive flow, elasto-plasticity and radiative transfer problems. At the following, Rannacher [16] studied this approach for incompressible viscous flows whereas Giles et al. [17] demonstrated the capability of this technique for drag and lift coefficients of a body immersed into a incompressible viscous flow. Based on the recent developments of the discontinuous Galerkin finite element method in fluid problems, several implementations of the adjoint-based error estimation are presented for inviscid [18,19] and turbulent viscous [20] flows. A comprehensive review of adjoint-based error estimation and mesh refinement methods in computational fluid dynamics for laminar and Reynolds-averaged Navier–Stokes applications is carried out by Fidkowski and Darmofal [21].

The adjoint equations are originally developed by Pironneau [22] and Jameson [23] for computation of derivatives of an output functional to be employed for gradient-based optimization methods. In order to solve the adjoint equations two different procedures, namely the continuous formulation and the discrete formulation, have been developed by researchers. In the continuous formulation the continuous form of the governing flow equations is first differentiated and then discretized, whereas the discrete formulation directly differentiates the discretized form of the governing equations. Although the continuous adjoint formulation has the advantage of less memory requirements, the implementation of the discrete formulation has been increased recently due to the simplicity of implementation and the unique capability of providing the exact discrete sensitivities. This simplicity comes from the fact that the transpose of the global Jacobian matrix, already calculated for the implicit solution of the flow equation, is utilized directly for the solution of the linear system of equations which arise from the discrete adjoint formulation. On the other hand, the treatment of the boundary conditions is quite straightforward in the case of the discrete formulation. A comprehensive comparison of the continuous and discrete adjoint formulations is presented in [24].

An important property related to the adjoint-based error estimation is adjoint consistency which ensures that the discrete adjoint problem is a consistent discretization of the continuous one. In the error estimation problems, an adjoint inconsistent discretization can lead to unsmooth or oscillatory adjoint solutions with discontinuity between elements that delivers adaptation in unnecessary areas leading to suboptimal convergence rate of functional estimates [25]. In particular, for the discretizations based on the high-order elements, enforcing adjoint consistency is essential for obtaining superconvergent functional estimates [26,27]. There has been a significant interest in studying the adjoint consistency in the context of finite difference method [28], stabilized continuous finite element method [27,29–33] and discontinuous Galerkin finite element method [25,34,35].

Regarding any central-based discretized formulation employed for the flow equations, it is a well established fact that the addition of stabilization terms to the original system of equations resulting from the discretization of the flow problem is essential to avoid the occurrence of numerical instabilities [36,37]. Within the family of the stabilization techniques, the so-called Finite Calculus (FIC) method has been successfully implemented for the stabilization of advective–diffusive transport and incompressible fluid flow problems [38–43]. Recently, a FIC-based stabilized formulation for the numerical solution of the compressible Euler and Navier–Stokes equations has been proposed in the context of Galerkin FEM by Kouhi and Oñate [44,45]. Since the systems of equations obtained from both the flow and the adjoint problems contain the same eigenvalues, the stabilization techniques developed for the flow equations can be implemented for solving the adjoint problem as well.

In this article, we extend the implementation of the FIC-FEM stabilized formulation presented in [44] to develop an adjoint-based error estimation and mesh adaptation framework for compressible inviscid flows. The system of equations obtained from the discretization of the flow problem is solved implicitly using a damped Newton's method benefiting from the exact Jacobian matrix proposed in [45]. At each iteration step, the inherent linear system resulting from the flow equations is solved with a preconditioned GMRES method. The transpose of the Jacobian matrix, already constructed from the discretized flow equations containing the FIC-based stabilization terms, is employed for the solution of the adjoint equations. In the paper, the adjoint consistency of the proposed stabilized method is investigated through checking smoothness and continuity of the adjoint solutions obtained for the presented examples.

AMR is performed here by using the local contributions of the functional error in conjunction with the classical h-refinement methodology, where each candidate element is divided into four by dividing each edge of the element into two. In order to demonstrate the capability of the FIC-FEM stabilized formulation in output-based error estimation problems, several examples are presented. By studying the quality of the results, it is found that the presented stabilized formulation provides enough stability for the numerical solution of the adjoint equations yielding an accurate estimation of the functional error during the AMR process.

The layout of the paper is the following: In Section 2 the compressible Euler equations along with the FIC-FEM stabilized formulation are described. Section 3 presents the derivation of the output-based error estimation and adaptive mesh refinement method using adjoint variables. The solution strategies for the flow and adjoint equations are explained in Section 4. The numerical results corresponding to the proposed error estimation and mesh refinement strategy for different output functional in subsonic, transonic and supersonic flow regimes are shown in Section 5. Finally, conclusions and general remarks are summarized in Section 6.

2. Flow problem formulation

2.1. Governing equations

The two-dimensional (2D) compressible Euler equations, including the mass balance, momentum and energy equations, are considered in this work and can be written in the following conservative form

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}_i}{\partial x_i} = \mathbf{0} \quad \text{for} \quad i = 1, 2 \tag{1}$$

where $\mathbf{U}$ and $\mathbf{F}$ are the vectors of conservative variables and inviscid fluxes, respectively, which can be expressed as

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