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# Guaranteed cost robust weighted measurement fusion steady-state Kalman predictors with uncertain noise variances

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ABSTRACT

Two classes of guaranteed cost robust weighted measurement fusion (WMF) one-step and multi-step Kalman predictors are presented by the Lyapunov equation approach for multisensor system with uncertain noise variances based on the minimax robust estimation principle. One class is to construct a maximal perturbation region of uncertain noise variances such that for all admissible perturbations in this region, the deviations of its actual accuracies with respect to the robust accuracy are guaranteed to remain within the prescribed range, and the maximal lower bound and minimal upper bound of accuracy deviations are given. The other class is to find minimal upper bound and maximal lower bound of accuracy deviations under given perturbation region of uncertain noise variances. The general and unified concept of guaranteed cost robustness is presented. The proof of the guaranteed cost robustness is presented by the Lyapunov equation approach. A simulation example shows the correctness and effectiveness of the proposed results.



## 1. Introduction

Multisensor information fusion has received great attention in recent years, and it has been applied to many fields including both military and nonmilitary applications such as target tracking, guidance, remote sensing, signal processing, GPS positioning, robotics, unmanned aerial vehicle (UAV) [1]. There exist two methodologies of information fusion based on Kalman filtering: the state fusion and measurement fusion [2,3]. The former combines the local state estimators to give a global optimal or suboptimal fused state estimate [4,5]. The latter can be classified as centralized measurement fusion and weighted measurement fusion, where the centralized measurement fusion has global optimality, but its disadvantage is to require a large computation and communication burden, while the weighted measurement fusion can give a global optimal fused state estimate by weighting the local measurement data, and only less computation burden is required. In order to compress the measurements of multisensor, based on the weighted least squares (WLS) approach, two optimal weighted measurement fusion (WMF) algorithms have been presented in [6,7].

Kalman filtering is a basic tool of multisensor information fusion. The optimal Kalman filtering requires to know the system model exactly. When there exist model reduction, parametric perturbations and unmodeled dynamics in the system model, an inexact model will degrade the filter performance, and may cause the filter to diverge. This has motivated many studies about robust Kalman filter (RKF) [8–15] and information fusion RKF [16–22] for systems with uncertainties of model parameters and/or noise variances. The so-called robust Kalman filter [8] is a filter such that for all admissible uncertainties, its actual filtering error variances are guaranteed to have a minimal upper bound. Up to now, many literatures of RKF focused on the systems with the model parameter uncertainties, the algebraic Riccati equations approach [8,9] and linear matrix inequality (LMI) approach [8,10] were used to solve this problem respectively. These researches on RKF have the limitations of assuming the uncertain model parameters but known noise variances, and only the maximal lower bound of accuracy deviations were given. The guaranteed cost robust Kalman filters for uncertain linear or nonlinear systems were presented in [11–13], where only the maximal lower bound of accuracy deviation were given; Using the game theory, based on the parameterization of the perturbations of noise variances, a guaranteed cost minimax RKF was presented for single sensor descriptor and non-descriptor systems with uncertain noise variances in the sense that there exists a maximal perturbation region of uncertain noise variances, such that for all perturbations in this region, the actual filtering

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accuracy deviations are guaranteed to be within the prescribed range [14,15]. Their limitation is only that the upper bound of the accuracy deviations is guaranteed. Recently, for multisensor systems with uncertain noise variances, applying the minimax robust estimation principle, based on the worst-case conservative system with the conservative upper bounds of noise variances, a unified weighted fusion minimax RKF theory was presented in [17–19], in which a Lyapunov equation approach was presented to prove the robustness of the proposed RKF, and the concepts of the actual accuracy and robust accuracy were proposed, and only the maximal lower bounds of accuracy deviations were given. The main difference between references [17] and [18] is that they were based on the different Lyapunov equations. Compared with [17], in [19], the augmented state approach was applied, so that the smoothing problem was converted into the filtering problem [17]. The multisensor system with uncertain cross-covariances of local estimation errors was considered in [20], while the multisensor system with uncertain noise variances was considered in [17–19]. So far, the general guaranteed cost robust information fusion filtering problem is not solved completely.

In this paper, the general and unified concept of guaranteed cost robustness is presented, which gives both the maximal lower bound and minimal upper bound of the accuracy deviations. Moreover, we present the two classes of guaranteed cost robust weighted measurement fusion (WMF) Kalman predictors for multisensor systems with uncertain noise variances. One class is to construct a maximal perturbation region of uncertain noise variances, for all admissible perturbations in this region, the accuracy deviations are guaranteed to be within the prescribed range, and both the minimal upper and maximal lower bound of accuracy deviations are given. The other class is that given the perturbation region of uncertain noise variances, to find the maximal lower bound and minimal upper bound of accuracy deviations over this region. Using the parameterization representation of the noise variance perturbations, the problem is converted into the nonlinear or linear program problem, which can be solved by the Lagrange multiplier method or the linear program (LP) method respectively. The proof of the guaranteed cost robustness is presented by the Lyapunov equation approach, which is different from that by the game theory in [14,15], and is different from the LMI approach and the algebraic Riccati equations approach [8].

This paper is organized as follows: the problem formulation is given in Section 2. Two classes of robust WMF guaranteed Kalman predictors are presented in Section 3. In Section 4, a simulation example is given to illustrate the correctness of the proposed results. The conclusions are presented in Section 5.

Notation: $R^n$ denotes the $n$-dimensional Euclidean space, $R^{n\times n}$ is the set of $n \times n$ matrix, $tr(\cdot)$ denotes the trace of a matrix, the superscript "T" denotes the transpose, $diag(\cdot)$ denotes the diagonal matrix.

## 2. Problem formulation

Consider the multisensor time-invariant system with uncertain noise variances.

$$x(t+1) = \Phi x(t) + \Gamma w(t) \tag{1}$$

$$y_i(t) = H_i x(t) + v_i(t), \quad i = 1, \cdots, L \tag{2}$$

where $x(t) \in R^n$ is the state to be estimated, $y_i(t) \in R^{m_i}$ and $v_i(t) \in R^{m_i}$ are the measurement and measurement noise of the $i$th subsystem, $w(t) \in R^l$ is the input noise, $\Phi$, $\Gamma$ and $H_i$ are known matrices with appropriate dimensions. $L$ is the number of sensors.

**Assumption 1.** $w(t) \in R^l$ and $v_i(t) \in R^{m_i}$ are mutually uncorrelated white noises with zero means and uncertain actual variances $\bar{Q}$ and $\bar{R}_i$, respectively, and $Q$ and $R_i$ being known conservative upper bounds of $\bar{Q}$ and $\bar{R}_i$, respectively, i.e.,

$$\bar{Q} \le Q, \qquad \bar{R}_i \le R_i \tag{3}$$

This means that

$$\Delta Q = Q - \bar{Q}, \quad \Delta Q \ge 0 \tag{4}$$

$$\Delta R_i = R_i - \bar{R}_i, \quad \Delta R_i \ge 0, \ i = 1, \cdots, L \tag{5}$$

**Assumption 2.** The uncertain noise variance perturbations $\Delta Q$ and $\Delta R_i$ can be parameterized as

$$\Delta Q = \sum_{i=1}^{p} \varepsilon_i Q_i \tag{6}$$

$$\Delta R_i = \sum_{j=1}^{q_i} e_j^{(i)} R_j^{(i)}, \quad i = 1, \cdots, L \tag{7}$$

where $\varepsilon_i \ge 0$, $e_j^{(i)} \ge 0$ are uncertain parametric perturbations, and the weighting matrices $Q_i \ge 0$ and $R_j^{(i)} \ge 0$ are known positive semi-definite symmetric matrices.

**Remark 1.** Specially, when $\Delta Q$ and $\Delta R_i$ are diagonal matrices, we can select $Q_i \ge 0$ as a diagonal matrix, whose $(i,i)$ element to be 1, and the other elements to be zeros. Similarly, we select $R_j^{(i)} \ge 0$ as a diagonal matrix with the $(j,j)$ element to be 1, and the other elements to be zeros. Hence we have

$$\Delta Q = diag(\varepsilon_1, \cdots, \varepsilon_l) = \sum_{i=1}^{l} \varepsilon_i Q_i,$$

$$\Delta R_i = diag\left(e_1^{(i)}, \cdots, e_{m_i}^{(i)}\right) = \sum_{j=1}^{m_i} e_j^{(i)} R_j^{(i)}, \quad i = 1, \cdots, L \tag{8}$$

Introduce the centralized fusion measurement equation

$$y_c(t) = H_c x(t) + v_c(t) \tag{9}$$

$$y_c(t) = \left[y_1^T(t), \cdots, y_L^T(t)\right]^T \tag{10}$$

$$H_c = \left[H_1^T, \cdots, H_L^T\right]^T \tag{11}$$

$$v_c(t) = \left[v_1^T(t), \cdots, v_L^T(t)\right]^T \tag{12}$$

where the fused measurement noise $v_c(t)$ has the conservative and actual variance matrix, respectively

$$R_c = diag(R_1, \cdots, R_L) \tag{13}$$

$$\bar{R}_c = diag(\bar{R}_1, \cdots, \bar{R}_L) \tag{14}$$

Defining $m_c = m_1 + \cdots + m_L$, and assuming $m_c \ge n$, then $m_c \times n$ matrix $H_c$ has full-rank decomposition [23–25]

$$H_c = M H_M \tag{15}$$

with partitioning $M$ as

$$M = \left[M_1^T, \cdots, M_L^T\right]^T, \quad M_i \in R^{m_i \times m} \tag{16}$$

and with $M \in R^{m_c \times m}$ having full column rank, and $H_M \in R^{m\times n}$ having full raw rank, and $m \le n$. Substituting (15) into (9), we have the WLS estimate of $H_M x(t)$ is given as [24,25]