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Boost-skipping trajectory optimization for air-breathing hypersonic missile

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ABSTRACT

To improve the range and penetration ability, a boost-skipping trajectory is proposed for the air-breathing hypersonic missile, with its scramjet ignited in break cycle mode. The longitudinal plane motion equations are established including the thrust term of lateral jets, and constraints on path parameters and terminal conditions are modeled. Using *hp*-adaptive Gauss pseudospectral method, the optimal control problem is transformed into a nonlinear programming problem with a series of algebraic constraints. And then adopting the sequential quadratic programming algorithm, the optimal trajectory is obtained satisfying the constraints. Several simulations and comparative cases are carried out, simulation results illustrate the proposed *hp*-adaptive pseudospectral method is superior to conventional optimization method on both computing efficiency and accuracy. The range of the missile is significantly enhanced in this optimized boost-skipping trajectory, which is 3.81 times longer compared with the existing level flight trajectory.

1. Introduction

The air-breathing hypersonic cruise missile, powered by scramjet, has become one of the newest weapons that are of great concern and development in major military powers. With the merits of without oxidizing agent, higher payload capacity, lower flight cost, and rapid global precision strike capability, this type of missile will be a new trend of cruise missile developing in the future [1]. After the air-breathing hypersonic missile launched from the aerial platform, the rocket motor boosts the missile to the realm of near space and the flight speed to 4–5 Ma. Then the scramjet ignites and the missile cruises in near space. Finally, the missile dives attack when approaching the target. The cruise phase is the longest stage of the whole trajectory, whereas the overload of the missile is so small that the maneuverability is limited. With the flight altitude and velocity are relatively stable, it could be captured and intercepted by air defense system.

Aiming at the limited penetration ability of the air-breathing hypersonic missile in cruise segment, a boost-skipping trajectory is designed in this paper with the missile controlled by both lateral thrust and aerodynamic force, which improves its overload and maneuverability in cruise segment. Boost-skipping trajectory was proposed by a German scientist Eugene Sanger in 1933, who aimed at a reentry hypersonic concept vehicle called 'Silverbird'. This trajectory could not only improve the missile penetration ability, but also significantly increased the missile range. It has been deeply researched on the trajectory design and optimization of the reentry hypersonic vehicle. The genetic algorithms [2,3] were applied to solve the optimal problem of RLV reentry trajectory. In Ref. [4], the direct parameter optimization method was conducted to reentry vehicle three-dimensional skipping trajectory optimization. Gauss pseudospectral method was also used to solve reentry trajectory optimization problems for Common Aero Vehicle (CAV) [5].

However, the research on boost-skipping trajectory optimization of air-breathing hypersonic cruise missile is not in-depth at present. Different from the reentry trajectory, the air-breathing missile is still in powered flight after the booster separation. The angle of attack is rigorously restricted by the operating condition of scramjet during the optimization process. Moreover, the thrust of scramjet is a time-varying value with the change of angle of attack, which results in more collocation points to satisfy the convergence condition. From another aspect, the existing air-breathing hypersonic missile could not flight in skipping trajectory due to the small overload in cruise phase, compared with the reentry vehicle. Therefore the aerodynamic force and lateral thrust blended control strategy with larger overload should be adopted to improve

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Nomenclature

C_x	aerodynamic drag coefficient	
C_y	lift coefficient	
F_N	thrust of the lateral jets	N
g	acceleration of gravity	m/s^2
h	flight altitude	km
I_{sp}	specific impulse of the scramjet	m/s
J	performance index	
L	range	km
Ma	Mach number	
m	mass	kg
N_0	initial number of collocation points	
N_c	computational number of collocation points	
n_y	normal overload	
P	thrust of the scramjet	N
$\dot{Q}$	heat flux	kW/m^2
q	dynamic pressure	Pa
R	mean radius of the earth	km
R_N	curvature radius at the stagnation point	m
s	aerodynamic reference area of the missile	m
t_{01}	the missile launch moment	s
t_{02}	the first ignition moment of scramjet	s
t_c	computation time	s
t_f	the end moment of skipping phase	s
t_{f1}	the end moment of boost phase	s
t_{f2}	the fuel exhaustion moment	s
u	control variable	
v	flight velocity	m/s
x	state variable	
α	angle of attack	deg
β	the throttle coefficient of the scramjet	
φ	pitch angle	deg
θ	trajectory tilt angle	deg
ρ	atmosphere density	kg/m^3

the mobility. Then the lateral thrust must be taken as a control variable, which increases the complexity of trajectory design.

The main methods currently solving trajectory optimization problems fall into two categories: the indirect method based on the maximum principle and the direct method on nonlinear programming theory [6]. The indirect method [7] eventually converts the optimal control problem into a two-point boundary value problem to gain the exact optimal solution, but it requires solving the state equations and co-state equations, which is computationally expensive for complex nonlinear dynamical equations. Meanwhile, the solving process is highly sensitive to the initial conjugate variables, and it is difficult to estimate and gain convergent solution. In contrast, the direct method [8] transforms the optimal control problem in continuous space into a nonlinear programming (NLP) problem using the parametric method. Then the optimal trajectory is derived by numerically solving this NLP problem. Moreover, the data-based framework [9,10] could be applied to handling optimal control system and process optimization. The pseudospectral method [11], which belongs to the direct method, shows great convergence in solving the trajectory optimization problem. This algorithm does not strictly require on the initial value, and can handle the optimal control problems containing path constraints effectively. However, the required collocation points argument with the increase of constraint conditions, so that the converted NLP problem could be more complicated which reduces the convergence speed and precision. To this issue, Darby [12] proposed the *hp*-adaptive Gauss pseudospectral method, which could adjust the number of collocation points and the order of interpolation polynomial adaptively, according to checking feasibility constrains of the state and control variables. Thus the speed and accuracy of optimization are improved greatly.

This paper aims at the trajectory optimization problem of the air-breathing hypersonic missile to improve its range and penetration ability. The trajectory optimization model was built, containing the constraints of angle of attack, the thrust of lateral jets, and ignition conditions of scramjet. A boost-skipping trajectory scheme with the aerodynamic force and lateral thrust blended control strategy was designed. Using *hp*-adaptive Gauss pseudospectral method, the optimal control problem was transformed into a nonlinear programming problem with a series of algebraic constraints. And then adopting the sequential quadratic programming (SQP) algorithm, the optimal trajectory was obtained satisfying the constraints.

2. Basic boost-skipping trajectory of hypersonic missile

The boost-skipping hypersonic missile combines the ballistic missile and the cruise missile, namely, using the characteristic of ballistic missile with a farther range and the merits of cruise missile with high precision and great maneuverability. The basic trajectory is shown in Fig. 1. The feasible flight scheme is described as follows: the rocket motor boosts the missile to tens of kilometers altitude. After booster separation, the cruise body in waverider configuration firstly gets through a skipping stage without power. When the speed and altitude of waverider dropping to the lowest ignition conditions, the scramjet ignites and the waverider skips in near space. The scramjet works in a break cycle ignition mode at this skipping trajectory to reduce the fuel consumption. After the scramjet runs out of fuel, the missile continues a new non-powered flight stage. When approach to the target, the missile enters into terminal trajectory and completes the dive attack.

For the basic trajectory in Fig. 1, the missile launch moment is denoted as t_{01}, where t_{f1} is the end moment of boost phase, t_{02} is the first ignition moment of scramjet, t_{f2} is the fuel exhaustion moment, and t_f is the end moment of skipping phase.

Since the terminal trajectory of the missile is associated with the target and is only a tiny fraction of the whole trajectory, the research in this paper is focused on trajectory optimization of boost and skipping phases, not including the terminal trajectory.

3. Longitudinal plane motion equations

Suppose the thrust of lateral jets is adjustable continuously. Assuming a non-rotating spherical earth with a symmetrical sphere, and the longitudinal plane motion equations with the blended control of the air-breathing hypersonic missile are:

$$\begin{cases} \dot{v} = \frac{1}{m}(P\cos\alpha - C_x qs) - g\sin\theta \\ \dot{\theta} = \frac{1}{mv}(P\sin\alpha + C_y qs + F_N\cos\alpha) - \left(\frac{g}{v} - \frac{v}{R+h}\right)\cos\theta \\ \dot{h} = v\sin\theta \\ \dot{L} = \frac{Rv\cos\theta}{R+h} \\ \dot{m} = -P/I_{sp} \\ \varphi = \alpha - L/R + \theta \\ n_y = (P\sin\alpha + C_y qs + F_N\cos\alpha)/(mg) \end{cases} \tag{1}$$

where m is the mass of the missile, v is the flight velocity, h is the flight altitude, L is the range, α is the angle of attack of the missile, φ is the pitch angle, θ is the trajectory tilt angle, P and F_N