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Stochastic optimal control for aircraft conflict resolution under wind uncertainty

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ABSTRACT

In this paper, a stochastic optimal control method is developed for determining three-dimensional conflict-free aircraft trajectories under wind uncertainty. First, a spatially correlated wind model is used to describe the wind uncertainty, and a probabilistic conflict detection algorithm using the generalized polynomial chaos method is proposed. The generalized polynomial chaos algorithm can quantify uncertainties in complex nonlinear dynamical systems with high computational efficiency. In addition, a numerical algorithm that incorporates the generalized polynomial chaos method into the pseudospectral method is proposed to solve the conflict resolution problem as the stochastic optimal control problem. The stochastic optimal control method is combined with the proposed conflict detection algorithm to solve the conflict resolution problem under the wind uncertainty. Through illustrative three-dimensional aircraft conflict detection and resolution examples with multiple heterogeneous aircraft, the performance and effectiveness of the proposed conflict detection and resolution algorithms are evaluated and demonstrated.

1. Introduction

The air traffic has been growing rapidly, and current air traffic management (ATM) system is under considerable stress. To accommodate the increasing air traffic, the International Civil Aviation Organization (ICAO) published a new operational concept of global ATM in 2005 [15]. The Next Generation Air Transportation System (NextGen) [10], the Single European Sky ATM Research (SESAR) [32], and the Collaborative Actions for Renovation of Air Traffic Systems (CARATS) [25] are currently ongoing programs in order to support the new era of air transportation in the United States, Europe, and Japan, respectively. These new ATM programs are aimed at harmonizing air traffic operations, reducing the heavy workload of air traffic controllers, and improving the safety, efficiency, capacity and environmental impact of the current ATM system. The primary concern of the ATM system is to guarantee safety, and one of the major safety critical situations is aircraft conflict when two or more aircraft experience a loss of the minimum allowed separation. The current conflict avoidance operations consist of two phases: conflict detection and conflict resolution. Aircraft trajectories are predicted to identify potential conflicts in the conflict detection phase, and conflict resolution strategies are provided to avoid the predicted conflicts in the conflict resolution phase. Most of the existing conflict detection and resolution algorithms can be categorized into two classes: deterministic and probabilistic approaches [18]. Since the accuracy of aircraft trajectory prediction is significantly influenced by various uncertainties such as wind, it is of vital importance to take into account the effect of wind uncertainty on conflict detection and resolution. However, it makes the problem more complicated and computationally intensive to consider the wind uncertainty. Therefore, in this study, we consider the conflict detection and resolution problem under the wind uncertainty and propose novel probabilistic conflict detection and resolution algorithms.

For the probabilistic conflict detection in the presence of uncertainty, the empirical distribution model of future aircraft positions [8,26,40], the dynamical model by using stochastic differential equations [14,28] and the probabilistic aircraft model based on the hybrid systems [4,21] are used to describe the aircraft motion. Using the probabilistic aircraft motion model, the conflict probability between aircraft is estimated to detect potential conflicts. In addition, the probabilistic conflict resolution problem is often formulated as a stochastic optimal control problem to deter-

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mine the optimal conflict resolution trajectory in the presence of uncertainty. In the previous works, Monte Carlo simulation [28], a Markov chain Monte Carlo framework [19] and Bayesian optimal design [17] are applied to determine an optimal control input. The stochastic optimal control problem is also solved by a Markov chain approximation and the Jacobi iteration [22].

In this study, the conflicts among multiple heterogeneous aircraft in the three-dimensional space, especially the terminal area, are considered, whereas most of the existing probabilistic conflict resolution methods [17,22,28] focus on the conflicts in the two-dimensional plane. As to the uncertainties during the flight in the previous works [4,8,19,26,28,40], various uncertainties such as the wind error, navigation errors and pilots' intents were studied, and the wind error was considered as the primary uncertainty because it has significant influence on the aircraft trajectory compared to other possible uncertainties. In this study, the wind prediction error, especially the spatially correlated wind error [4,7,14], is considered because the wind correlation usually has a significant effect on the trajectories of aircraft which are close to each other, and therefore conflict detection and resolution [4]. Unlike the two-dimensional aircraft dynamics in the previous works mentioned above, the three-dimensional aircraft dynamics containing the spatially correlated wind error are much more complex because of the high-dimensional state space and complicated stochastic models. In addition, nonlinear optimal control problems for such complex dynamical systems with the high-dimensional state space are challenging to solve, and require sophisticated optimization approaches. In this paper, we propose novel probabilistic conflict detection and resolution algorithms by employing the generalized polynomial chaos (gPC) method [37–39], which can quantify uncertainties in the complex nonlinear dynamical systems with high computational efficiency. To detect potential conflicts, the conflict probability between aircraft is estimated by the probabilistic conflict detection algorithm. For the conflict resolution problem, we apply the pseudospectral method [27] which is a recently developed numerical method to solve deterministic nonlinear optimal control problems. A numerical algorithm incorporating the gPC method into the pseudospectral method is proposed to deal with stochastic elements and solve the stochastic optimal control problems. The stochastic optimal control method is combined with the probabilistic conflict detection algorithm to guarantee the resolution of potential conflicts under the wind uncertainty.

The paper is organized as follows. Section 2 presents the probabilistic conflict detection and resolution algorithms employing the gPC method. In Section 3, the conflict detection and resolution problem with multiple heterogeneous aircraft is formulated and solved. Through numerical simulations, the effectiveness and performance of the probabilistic conflict detection and resolution algorithms are evaluated and demonstrated. Finally, conclusions are provided in Section 4.

2. Probabilistic conflict detection and resolution algorithms

In this section, we first introduce the stochastic aircraft dynamics including spatially correlated wind uncertainty, and propose a conflict detection algorithm based on the gPC method. After that, a stochastic optimal control method incorporating the gPC algorithm into the pseudospectral method is developed to solve the conflict resolution problem.

2.1. Stochastic aircraft dynamics

We consider the conflicts between aircraft in the three-dimensional terminal airspace in which the aircraft coming from different directions merge to the final approach fix to be aligned for landing.[1] We assume that the aircraft descends to the merging point following the continuous descent approach (CDA) profile, i.e., the aircraft descends continually at idle thrust from cruise to landing [6]. The aircraft dynamics are given by the following point mass model with six state variables $\boldsymbol{x} = (x, y, h, v, \psi, \gamma)^T$ and two control variables $\boldsymbol{u} = (C_L, \phi)^T$:

$$\dot{x} = \frac{v\cos\psi\cos\gamma + w_x}{3600} \tag{1}$$

$$\dot{y} = \frac{v\sin\psi\cos\gamma + w_y}{3600} \tag{2}$$

$$\dot{h} = \frac{1519}{900} v\sin\gamma \tag{3}$$

$$\dot{v} = \frac{900}{463}\left(\frac{T_{\text{idle}}}{m} - g\sin\gamma\right) - \frac{1519}{1800}\frac{\rho v^2 S C_D}{m} \tag{4}$$

$$\dot{\psi} = \frac{1519}{1800}\frac{\rho v S C_L \sin\phi}{m\cos\gamma} \tag{5}$$

$$\dot{\gamma} = \frac{1519}{1800}\frac{\rho v S C_L}{m} - \frac{900}{463}\frac{g}{v}\cos\gamma \tag{6}$$

where x and y are the Cartesian coordinates in nautical miles (nmi); h is the altitude in feet (ft); v is the true airspeed in knots (kt); ψ is the heading angle; γ is the flight path angle; C_L is the aerodynamic lift coefficient; ϕ is the bank angle; ρ is the air density in pounds per cubic foot (lb/ft^3); g is the acceleration of gravity and set to 9.807 m/s^2; and w_x and w_y are the stochastic wind velocities in knots (kt) in the x and y directions, respectively. It is reasonable to assume that the vertical component of the wind w_h is set to zero for simplicity because w_h is relatively small compared to the horizontal components w_x and w_y, excluding particular situations such as wind shear [4]. The idle thrust T_{idle} in pounds-force (lbf), the aerodynamic drag coefficient C_D, the wing area S in square feet (ft^2) and the aircraft mass m in pounds (lb) are obtained from the Base of Aircraft Data (BADA) model [9]. m is assumed to be constant because the fuel consumption is negligible since the time interval considered for conflict detection and resolution is short. Note that the constant values in Eqs. (1)–(6) are the unit conversion factors.[2]

As to the wind uncertainty, the wind model contains the deterministic and stochastic components [4,14]. In this study, the deterministic component representing the meteorological prediction is ignored and set to zero for simplicity. The wind model accounts for only the stochastic component, i.e., the wind prediction error representing the uncertainty in the deterministic meteorological prediction. Thus, the wind velocities w_x and w_y are referred to the wind prediction errors. In this study, the wind error is assumed to be time-invariant, because the time horizon for conflict detection and resolution considered in this study is short (approximately 10 min) and the temporal change in the wind error is small [22]. To describe the wind errors more realistically, the spatially correlated wind model is considered. From the correlated wind model [4], which is constructed based on the comparison between the real historical aircraft reports and wind forecast data [7], $w_x(x, y, h)$ and $w_y(x, y, h)$ are assumed to be Gaussian random processes with zero mean and the following exponential covariance function:

$$\begin{aligned} C\left((x, y, h), (x', y', h')\right) &= \sigma_w^2 \exp\left(-\mu_x|x - x'|\right)\exp\left(-\mu_y|y - y'|\right) \\ &\quad \times \exp\left(-\mu_h|P_A(h) - P_A(h')|\right) \end{aligned} \tag{7}$$

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[1] Note that though terminal airspace operations are considered for illustration, the proposed algorithm can be applied to any phase of flight.

[2] Note that 1 nmi = 1852 m ≈ 6076 ft.