Aerospace Science and Technology ••• (••••) •••–•••

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Aerospace Science and Technology

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Elastoplastic buckling analysis of thin-walled structures

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ARTICLE INFO

Article history:
Received 21 July 2014
Accepted 4 March 2015
Available online xxxx

Keywords:
Elastic–plastic buckling
Kantorovich technique
Buckling of stiffened plates
Ramberg–Osgood law
Buckling paradox

ABSTRACT

The study is concerned with the elastoplastic buckling of thin-walled beams and stiffened plates, subjected to in-plane, uniformly distributed, uniaxial and biaxial load. The ruling differential equations have been solved analytically by using the Kantorovich technique and the obtained displacement field has been employed in a general procedure that, by using the framework derived by the finite element method, is able to analyze the elastoplastic buckling behavior of prismatic beams and stiffened plates with arbitrary cross-section. The inelastic effect is modeled through a stress–strain law of the Ramberg–Osgood type, and both the incremental deformation theory and the J_2 flow theory are here considered. The reliability of the numerical procedure is illustrated for rectangular plates, and the contradicting results obtained by using the two plastic theories are discussed in detail. Finally, the performance of the method is illustrated through the analysis of a C-section and five different closed section columns.

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1. Introduction

Due to their relevance in the design of thin structures, the critical behavior of plates, both flat and stiffened, has been extensively studied in the past years, as documented by the number of text [1,2] and papers [3,4] on the issue.

More recently, a number of authors have also investigated the effect of elastoplastic constitutive models on the prediction of the buckling load of thin structures, generally by employing models based on both the incremental J_2 deformation theory (DT) and the J_2 flow theory (FT). Starting from the pioneering works of – among others – Handelmann and Prager [5], Bijlaard [6] and Bleich [7] in the 1950s, different aspects of buckling behavior in elastoplastic range have been analyzed, and collected in papers dedicated to the analytical investigation of buckling [8] and post-buckling [9] of simple structures. Some of these works examine the causes of the "plastic buckling paradox", as it was first revealed in a series of theoretical investigations on the buckling of a simply supported flat plate of infinite length, through both the FT [5] and the DT [6]. A comparison of the results shows that "the buckling load obtained from the "more rational" FT model always far exceeds the experimental observation, whereas the results obtained adopting the "simplest", total strain DT, predict buckling loads lower than FT model and in any case in closest agreement with those obtained experimentally. Moreover, the differences between the two models increase as the level of plasticity increases, giving in some cases results with different order of magnitude" [10].

As reported in [8], such aspects drastically affect the behavior and the load carrying capacity of flat plates subject to uniaxial or biaxial in-plane load, regardless of the boundary conditions. For instance, Durban and Zuckerman found, for specified compression/tension ratios, an optimal loading path for the DT that has no correspondence in FT. The works of Wang et al., which analyze the elasto-plastic buckling of thin [11] and thick [12] flat plates by differential quadrature method, confirm the results obtained in [8] and show that the paradox also involves flat plates of different geometry, such as triangular or elliptical, with different boundary conditions. A comparison of incremental DT and flow rule has been recently proposed in [13], where a new algorithm, based on a Generalized Differential Quadrature (GDQ) discretization technique to solve the out-of-plane plate equation, has been presented.

The aim of this work is to present a new analytical formulation to define the critical behavior of both flat and stiffened plates, as well as of thin walled columns with open or closed cross sections, and characterized by non-linear elastoplastic behavior. To this end a systematic procedure based on a finite element algorithm capable of accounting for both local and global phenomena has been developed. As they are analytical, the obtained results do not depend on the discretization adopted, and reliable solutions can be achieved with the minimum number of elements required to represent the structural geometry, with the double effect of stabilizing the numerical procedure and accelerating the computing time required for the eigenvalue and eigenvector analysis.

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http://dx.doi.org/10.1016/j.ast.2015.03.001
1270-9638/© 2015 Published by Elsevier Masson SAS.

Please cite this article in press as: E. Ruocco, Elastoplastic buckling analysis of thin-walled structures, Aerosp. Sci. Technol. (2015), http://dx.doi.org/10.1016/j.ast.2015.03.001

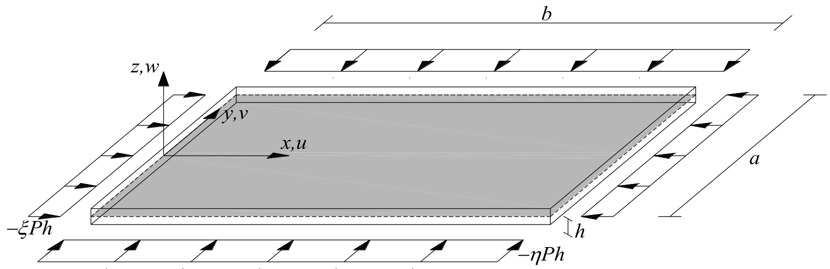

Fig. 1. Geometry and load condition of a rectangular plate.

For validation purposes, the proposed procedure has been applied to flat plates, and the results compared to the analytical and/or numerical solutions available in literature. In such a preliminary analysis particular attention is paid to the evaluation of the instantaneous elastic moduli, and to the differences between them deriving from the two models adopted. The analysis indicates that the (correct) use of the constant value of Lamè modulus G in FT shows, as consequence, an irregular behavior of the corresponding generalized elastoplastic modulus which, when the ratio between the in-plane loads is greater than a critical value, in plastic range increases its stiffness, in contrast to what happens to the other modulus. Having adopted the secant modulus G_s, the corresponding modulus calculated in the DT does not present the same anomaly. Observing that just in correspondence of such critical ratio the buckling path corresponding to the two theories begins to diverge, it is possible to assert that the irregular behavior of the elastic modulus related to the G-coefficient is a cause, or at least a concomitant cause, of the plastic buckling paradox.

Another aspect considered in this work is the adequacy of the von Kármán hypothesis in the negligible contribution of second-order strain terms related to in-plane displacement. In a previous work devoted to the influence of such terms on the elastic buckling of shell and both flat and stiffened plates [14], the authors demonstrate how, when the critical mode involves comparable in-plane and out-of-plane displacement, the omission of the non-linear strain terms related to in-plane displacements can considerably overestimate the critical load. If such cases never happen for flat plates, for which the von Kármán simplification is thus correct, it is quite common for stiffened plates and prismatic beams, when buckling mode involves torsional, flexo-torsional or global flexural buckling displacement. In the present work is – to the author's knowledge for the first time – the influence of such non-linear terms on the elasto-plastic buckling of stiffened plates is presented and discussed.

In order to capture the inelastic effect, the constitutive behavior is here modeled through a stress–strain law of the Ramberg–Osgood type that, with the basic equations ruling the elastoplastic buckling of Kirchhoff plates subjected to uniaxial or biaxial compression and under both the FT and DT hypothesis, are introduced in the following section. Section 3 reports an analytical solution based on the Kantorovich technique, and its generalization in a FEM-like procedure for the buckling analysis of structures ascribable to plates rigidly connected together along their edges. The same section includes the numerical procedure necessary to solve the eigenproblem associated to the corresponding elastoplastic stiffness matrix. In Section 4 parametric analyses related to a single plate with different boundary and load conditions, a simply supported open C-section and five different thin-walled closed section beams are reported and discussed. The last part of the paper contains some conclusive considerations.

2. Governing equations

Consider a rectangular isotropic plate of dimension (a, b) and uniform thickness h, schematically represented, together with the local reference system adopted in this work, in Fig. 1. The plate is subjected to biaxial in-plane compressive load $n_x = -\xi Ph$ and $n_y = -\eta Ph$, with (ξ, η) fixed ratio parameters, such that $(\xi = \eta = 1)$ describes equibiaxial compression, $(\xi = 1, \eta = 0)$ uniaxial compression in x direction, and so on. In plane stress conditions the constitutive behavior can be defined as follows:

$$\begin{bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\tau}_{xy} \end{bmatrix} = E \begin{bmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{xy} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{xz} & \alpha_{yz} & \alpha_{zz} \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \\ \dot{\gamma}_{xy} \end{bmatrix} \tag{1}$$

where E is the elastic Young modulus and α_{ij} are instantaneous moduli depending on the plasticity theory considered to model material behavior. Here two plasticity theories are considered, namely the incremental, or flow, theory of plasticity, with the Prandtl–Reuss constitutive equation:

$$\dot{\sigma}_{ij} = 2G\dot{\varepsilon}_{ij} + \lambda\delta_{ij}\dot{\varepsilon}_{kk} - 3(G - G_t)\frac{S_{ij}S_{kl}\dot{\varepsilon}_{kl}}{\sigma_e^2} \tag{2}$$

and the deformation theory of plasticity with the Hencky constitutive relation:

$$\dot{\sigma}_{ij} = 2G_s\dot{\varepsilon}_{ij} + \lambda_s\delta_{ij}\dot{\varepsilon}_{kk} - 3(G_s - G_t)\frac{S_{ij}S_{kl}\dot{\varepsilon}_{kl}}{\sigma_e^2} \tag{3}$$

In Eqs. (2) and (3) $(G, \lambda, G_s, \lambda_s)$ are the elastic and the secant Lamè coefficients, G_t is the tangent shear modulus, S_{ij} are the stress deviator components and σ_e is the equivalent stress that, for the load conditions considered, assumes the value:

$$\sigma_e = P\left(\xi^2 - \xi\eta + \eta^2\right)^{\frac{1}{2}} \tag{4}$$

For both the FT and DT models the instantaneous moduli α_{ij} can be expressed in the form:

$$\begin{bmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{xy} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{xz} & \alpha_{yz} & \alpha_{zz} \end{bmatrix} = \frac{1}{\rho}\begin{bmatrix} c_{yy}c_{zz} - c_{yz}^2 & c_{xz}c_{yz} - c_{xy}c_{zz} & 0 \\ & c_{xx}c_{zz} - c_{xz}^2 & 0 \\ symm & & c_{xx}c_{yy} - c_{xy}^2 \end{bmatrix} \tag{5}$$

where:

$$c_{xx} = 1 - 3\left(1 - \frac{E_t}{\bar{E}}\right)\frac{s_y^2}{4}, \qquad c_{xz} = 0,$$

$$c_{yy} = 1 - 3\left(1 - \frac{E_t}{\bar{E}}\right)\frac{s_x^2}{4}, \qquad c_{yz} = 0,$$

$$c_{xy} = -\frac{1}{2}\left(1 - (1 - 2\nu)\frac{E_t}{E} - 3\left(1 - \frac{E_t}{\bar{E}}\right)\frac{s_x s_y}{2}\right),$$

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