Aerospace Science and Technology ••• (••••) •••–•••

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

# Modeling and analysis of dynamics for spacecraft relative motion actuated by inter-satellite non-contacting force

Yuan-wen Zhang *, Le-ping Yang, Yan-wei Zhu, Huan Huang, Wei-wei Cai

*National University of Defense Technology, Changsha, Hunan 410073, PR China*

ARTICLE INFO

*Article history:*
Received 1 February 2013
Received in revised form 29 April 2013
Accepted 5 March 2015
Available online xxxx

*Keywords:*
Spacecraft relative motion
Inter-satellite non-contacting force
Unified modeling
Dynamic model
Dynamics analysis

ABSTRACT

With the recent flurry of research on spacecraft relative motion actuated by inter-satellite non-contacting force, i.e., electrostatic force, electromagnetic force and magnetic flux-pinned force, the need has become apparent for dynamic modeling and dynamics analysis that capture the characteristics of this generic force. Based on the basic derivation of electrostatic force model and some theorems, the actuations of electrostatic force, electromagnetic force/torque and magnetic flux-pinned force/torque are unified and the corresponding mathematic models are derived. And then, with a general notation of this inter-satellite non-contacting force, the dynamic models are developed with respect to several assumptions of circular orbit, elliptical orbit and general Keplerian orbit pertaining to the motion of center of mass of the spacecraft cluster. A detailed dynamics analysis based on these derived models are carried out, including constraints of the relative motion dynamics, the special characteristics of applications to the near-Earth relative orbit motion and the deep space relative motion, and then a case with co-rotating pair of satellites subject to the previously mentioned inter-satellite non-contacting forces was simulated to validate the dynamic modeling and dynamics analysis. At last, some useful conclusions follow.

## 1. Introduction

The actuation force for spacecraft operation can be classified to three categories: the independent thruster force [1] acting on individual spacecraft, the inter-satellite contacting force [2,3] and the inter-satellite non-contacting force [4–7] acting on all the relevant spacecraft. Compared to the first actuation force, the third one has advantages of fuel-efficiency, cleanness, simultaneous and distant actuation. Specifically, as to the spacecraft cluster missions, the relative motion between spacecraft, instead of the absolute inertial motion in space, is important. However, thruster force actuates inertial degrees-of-freedom, while the inter-satellite non-contacting force actuates relative degrees-of-freedom. Compared to the second actuation force, the third one has advantages of higher safety, more degrees-of-freedom for relative motion and mission operations. Therefore, the inter-satellite non-contacting force has attracted more and more attention in the research field recently.

Nowadays, the three mainly studied inter-satellite non-contacting forces are the electrostatic force, the electromagnetic force and the magnetic flux-pinned force. Although their actuation principles are different, it is actually true that they share several similar characteristics. For example, they all belong to field force, and being narrow range but high control precision. Schweighart has proven that the far-field electromagnetic force/torque models are equal to the mathematical models produced by two sets of charge, each of which includes a $+q$ and a $-q$ separated by a distance [8]. In addition, Wilson et al. [9], Jones and Peck [10] have verified that the actuation effect of magnetic flux-pinned force could be empirically represented by the interactions of the actual magnetic field with the "frozen magnetic dipole" created in response to the field source's position at the time of cooling and the "mobile magnetic dipole" that changes position as the actual magnetic field source moves. Therefore, based on this representation and the electrostatic/magnetostatic duality theorem, there is a possibility of unifying the mathematical modeling of the three forces.

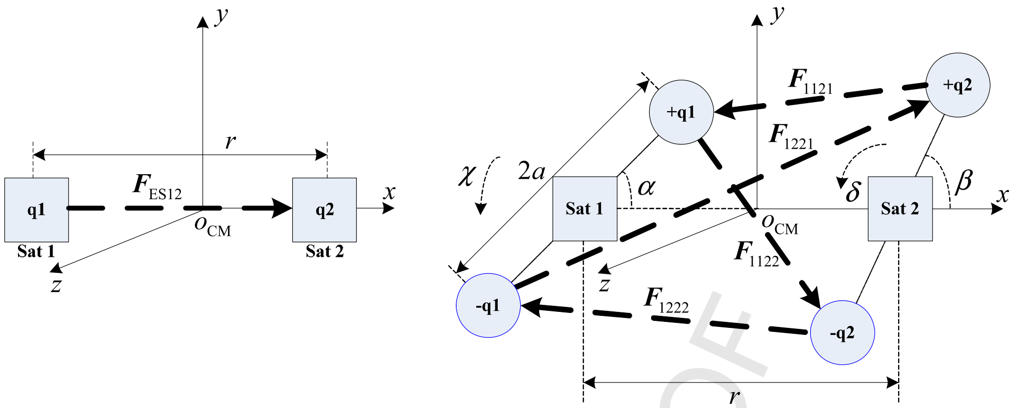
With the development of space operation technology, such as formation flight, rendezvous and docking, fractionated spacecraft and on-orbit servicing, relative motion dynamics of spacecraft has become a hot research topic and the main mission implementation mode. To date, many modeling approaches of the spacecraft relative motions enabled by inter-satellite non-contacting force are based on the Hill equations, which are linear and convenient for dynamics analysis and controller design [11]. However, the Hill equations are only valid for circular reference orbit and small relative distance between the spacecraft, which poses limitations on the applications to other kinds of conic-section reference orbit mo-

---

* Corresponding author. Tel.: +86 13637479241.
*E-mail address:* pointmaker8384@126.com (Y.-w. Zhang).

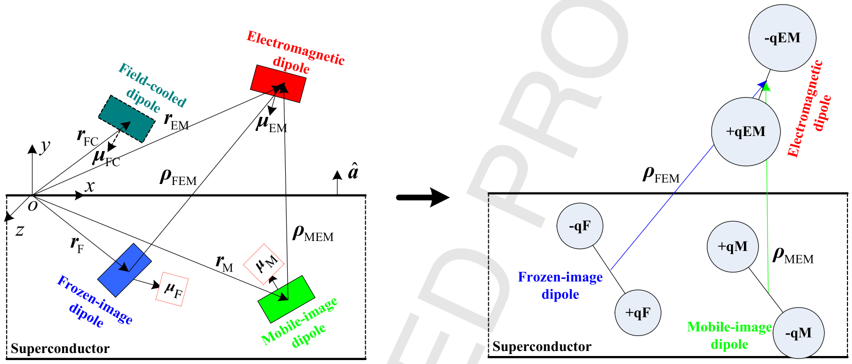
http://dx.doi.org/10.1016/j.ast.2015.03.003

Please cite this article in press as: Y.-w. Zhang et al., Modeling and analysis of dynamics for spacecraft relative motion actuated by inter-satellite non-contacting force, Aerosp. Sci. Technol. (2015), http://dx.doi.org/10.1016/j.ast.2015.03.003



(a) Electrostatic force

(b) Electromagnetic force



(c) Magnetic flux-pinned force

**Fig. 1.** General formulation of the three inter-satellite non-contacting forces.

tion. In addition, the relative attitude dynamics should also be exploited.

As these inter-satellite non-contacting forces have found more and more applications, some novel and interesting dynamics characteristics appear [12–16], which constrain the degrees-of-freedom of spacecraft relative motion. However, these dynamics characteristics have not been generally and systematically studied by far.

In this paper, based on the unified modeling of the three inter-satellite non-contacting forces, several dynamic models and dynamics analysis of spacecraft relative motion are theoretical derivation and numerical validated. Firstly, applying the electrostatic/magnetostatic duality theorem and the image-dipole representation of the magnetic flux-pinned force, the unified mathematical models of the three forces are derived. Secondly, considering several different orbit motion assumptions of the center of mass (denoted as CM) of the spacecraft cluster, including circular orbit, elliptical orbit and general Keplerian orbit, based on the equations of the Hill model, the T–H model and the orbit elements model, three corresponding relative motion dynamic models actuated by this generic inter-satellite non-contacting force are developed. Thirdly, after investigating these derived models, the constraints of relative motion dynamics, the characteristics of application to the near-Earth relative orbit motion and the deep space relative motion are analyzed in detail. Fourthly, cases with co-rotating pair of satellites subject to the three inter-satellite non-contacting forces were simulated to validate the dynamic modeling and dynamics analysis. At last, some useful conclusions are given.

## 2. Dynamic modeling

In this section, the mathematical models of the three inter-satellite non-contacting forces are unified, and several dynamic models of relative translational motion considering different referenced orbit assumptions are derived. In addition, the dynamic model of relative attitude motion is simply put forward.

### 2.1. General formulation of the three inter-satellite non-contacting forces

To deal with these dynamic problems in a unified way, we try to develop a general mathematical model for these forces. In general, the electrostatic force has the simplest formulation with respect to the other two forces, so this general mathematical model is developed on the basis of electrostatic formulation. The electrostatic force formulation for two satellites is depicted as Fig. 1(a). Based on the electrostatic/magnetostatic duality theorem [8], we utilize the electrostatics duality approach to derive the far-field electromagnetic force/torque models, depicted as Fig. 1(b). Magnetic flux-pinned force refers to the interaction between a magnetic field and a high-temperature superconductor (HTSC), which establishes an equilibrium position and orientation of the magnetic field relative to the HTSC. Based on the image-dipole model of magnetic flux-pinned force [10,17,18] and the electrostatic/magnetostatic duality theorem, we derive the multiple electrostatic forces formulation for the magnetic flux-pinned interaction, depicted as Fig. 1(c).

Therefore, based on the depictions of Fig. 1, the general formulations of the three inter-satellite non-contacting forces are derived as follows.

For electrostatic force, ignoring the Debye shielding effect, only the axis aligned force exists, which can be derived as

$$\boldsymbol{F}_{\mathrm{ES}12} = -k_c \frac{q_1 q_2}{r^2} \hat{\boldsymbol{r}} \tag{1}$$

where $\boldsymbol{F}_{\mathrm{ES}12}$ denotes the electrostatic force on satellite 1 actuated by satellite 2, Coulomb constant $k_c = 8.99 \times 10^9\ \mathrm{Nm}^2\,\mathrm{C}^{-2}$, $r$ is the

Please cite this article in press as: Y.-w. Zhang et al., Modeling and analysis of dynamics for spacecraft relative motion actuated by inter-satellite non-contacting force, Aerosp. Sci. Technol. (2015), http://dx.doi.org/10.1016/j.ast.2015.03.003