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A new approach to autonomous rendezvous for spacecraft with limited impulsive thrust: Based on switching control strategy

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ABSTRACT

This paper proposes a novel impulsive control approach for autonomous rendezvous of spacecraft with limited thrust. Based on the dynamic model established by Clohessy–Wiltshire (C–W) equations, the rendezvous process is regarded as impulse phase and free motion phase which are considered as closed-loop system and open-loop system respectively. The durations of the two phases are scaled by sampling period. Then, based on Lyapunov theory, two virtual energy functions of the subsystems are introduced, and the rendezvous problem is regarded as an asymptotic stabilization problem of the switching system. Meanwhile, the thrust limitation is also taken into consideration. The impulsive closed-loop control problem is finally transformed into a convex optimization problem and the calculation steps are proposed. With the designed controller, the impulsive thrust during the rendezvous is calculated at the impulse instants. Furthermore, the calculated thrust is kept below a given bound, which can also be minimized according to specific requirements. The effectiveness of the proposed approach is illustrated by simulation examples.

1. Introduction

In recent years, autonomous spacecraft rendezvous is widely regarded as an essential problem for many astronautical missions such as spacecraft repairing, saving, intercepting, etc. The relative problems attract more and more attention in the world.

Generally, there are two kinds of orbital control methods according to different thrust patterns: continuous thrust and impulse thrust. With continuous control, the needed thrust is provided continually, and the chaser spacecraft is kept under control during the whole rendezvous process. Thus, precise transfer orbit can be obtained by designing proper continuous thrust strategies according to specific requirements. Many correlative results have been reported in recent years. For example, adaptive control method is adopted for continuous thrust controlled rendezvous process in [13,15,27]. Based on the continuous relative positional error, five control strategies are proposed by using Lyapunov theory and Matrosov's theorem in [14]. Furthermore, many kinds of constraints during the rendezvous process have been studied based on continuous control method. In [4] and [6], the sight measuring constraint and the collision avoidance constraint are studied respectively. The relative motion control problems with limited-power or low-thrust are studied in [18,19,28,31]. To minimize the thrust during the rendezvous, a property called "null controllability with vanishing energy (NCVE)" was proposed for the C–W equations recently, and some effective correlative results have been reported in [1,7,26].

Nevertheless, we should note that, although continuous control has advantage to enhance the orbital accuracy, it may also brought worse fuel economy for the rendezvous process. And more crucially, it is difficult to produce absolute continuous and precise variable thrust for the thrusters in practice. Actually, the continuous control strategies are usually approximately realized by multiple impulse thrust with short constant interval or multiple impulse with constant velocity increment. Thus, the continuous control methods are actually just some special impulsive methods in practice. Therefore, the research on impulsive control method is more significant for practical rendezvous engineering.

Different from the analysis of continuous control method, the chaser spacecraft is not always under control during the rendezvous process for the impulsive control method. Based on the analytic solution of the C–W equations, the impulsive control strategy can be designed according to specific conditions such as fixed rendezvous time or fixed terminal condition. For instance, the multi-impulse rendezvous problems with fixed time are studied in [20,21], and the two-impulse rendezvous problem with fixed

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terminal condition is studied in [9]. Furthermore, for the problem with both fixed time and fixed terminal condition, [2,3] investigate the necessary and sufficient condition of the optimal impulse method, and [22] proposes an optimal two-impulse method for this problem. The optimal solutions of these works are obtained by solving the two-point boundary problem, which is also called Lambert problem in aerospace field. Recently, many other optimal methods have been adopted to deal with the impulse rendezvous problem. In [16,17], the multi-objective genetic algorithms (MOGAs), non-dominated sorting genetic algorithm (NSGA-II) and simulated annealing (SA) have been adopted for designing the optimal impulse control strategy for spacecraft rendezvous; in [8], the NCVE property of the C–W equations has also been extended to solve the impulse relative orbital transfer problem.

However, it should be noted that, most of the existing impulse control strategies depend on the exact prior knowledge of the relative states between two spacecraft during the rendezvous process, and the impulse series is previously set before the rendezvous mission starts. Thus, these methods are actually open-loop control methods, which are easily effected by disturbance or other uncertain factors. Besides that, solving two-point boundary problem or any calculation of MOGAs, NSGAs or SAs needs lots of numerical computations. Thus, designing an impulse feedback control method which can determine the needed impulse thrust at the impulse instant during the rendezvous process is significant for practical rendezvous engineering, which motivates our work in this paper.

By analyzing the rendezvous process with impulsive thrust, we found that the whole rendezvous process contains two different phases, which can be called "impulse phase" and "free motion phase". After every impulse action, the chaser spacecraft moves freely during the free motion phase until next impulse action. Thus, the whole rendezvous process can be regarded as a series of alternations between these two phases. Then, a question emerges: if these two phases are regarded as two subsystems, whether the rendezvous problem can be solved by considering the rendezvous process as a switching system? After careful studying on the correlative problems, we found that this idea is feasible. Thus, in this paper, a novel impulsive control approach, which is based on the analysis of switching system, is proposed for solving the spacecraft autonomous rendezvous problem.

By introducing a state feedback control for the impulse phase, the needed thrust at the impulse instant can be determined according to the relative state between the chaser and the target. Thus, the relative motion system during the impulse action can be regarded as a closed-loop system. Correspondingly, the relative motion system during the free motion phase can be regarded as an open-loop system. With these two subsystems, and based on Lyapunov theory, the autonomous rendezvous problem is regarded as an asymptotic stabilization problem of a switching system. To facilitate the analysis, a discrete view of the relative motion is employed, and the durations of the two subsystems are scaled by the sampling period. Actually, impulsive control in discrete time for switching system have been widely studied for many other kinds of control applications [10,23,24,30,33]. Thus, the impulsive control strategy based on switching system is studied for autonomous spacecraft rendezvous problem. In addition, the thrust constraint is also considered due to the fact that it is impossible to produce the unlimited thrust in practice.

Based on above analysis, in this paper, the stability of the switching system and the thrust constraint are analyzed by introducing two virtual energy functions of the subsystems, and the controller design problem is transformed into a convex optimization problem, which has been proven effective for solving many kinds of control problems [11,12,25,29,32]. Different from other studies on impulsive control problem for spacecraft rendezvous, this paper considers the impulse action phase and free motion phase separately and a novel feedback control approach based on a switching system is proposed. Thus, some other correlative analysis of switching system can be adopted to study the impulsive rendezvous problem based on the results of this paper, and the proposed approach can be extended to deal with other more specific requirements.

Notations. The notation used throughout the paper is fairly standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ denotes the set of all $n \times m$ real matrices; $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. For a real symmetric matrix W, the notation $W > 0$ ($W < 0$) is used to denote its positive- (negative-) definiteness. $diag\{\ldots\}$ stands for a block-diagonal matrix. In symmetric block matrices or complex matrix expressions, we use an asterisk ($*$) to represent a term that is induced by symmetry. I and 0 denote the identity matrix and zero matrix with compatible dimensions. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Impulse control problem for spacecraft rendezvous

In this section, the dynamic model is established based on C–W equations. Then, by considering the properties of the impulsive orbital control, the discrete view is adopted. Then, open-loop control and closed-loop control for the rendezvous process are considered respectively. Finally, by considering the thrust constraint, the problem we study in this paper is formulated.

2.1. Dynamic model

The most basic model in spacecraft rendezvous research is given by Clohessy–Wiltshire (C–W) equations [5], which have been widely adopted to study the spacecraft relative motion problems. By assuming circular target's orbit and small distance between the chaser and the target, the linearized equations of the relative motion between them can be described as

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = \frac{1}{m}T_x, \\ \ddot{y} + 2n\dot{x} = \frac{1}{m}T_y, \\ \ddot{z} + n^2 z = \frac{1}{m}T_z, \end{cases} \tag{1}$$

where x and y are the radial component and along-track component of the chaser's position relative to the target respectively, z is the out-plane component which completes the right handed coordinate system, n is the mean motion of the target, m is the mass of the chaser, T_i ($i = x, y, z$) is the ith component of the control thrust. Then, by defining the state vector $\mathbf{x}(t) = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$, input vector $\mathbf{u}(t) = [T_x, T_y, T_z]^T$ and the proper matrices A and B which can be readily obtained according to (1), the relative motion equations in (1) can be written as the following state space function

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t). \tag{2}$$

Obviously, the rendezvous process can be regarded as the asymptotic stabilization process of the system in (2), and the main task of the studies on rendezvous is to determine the proper control input $\mathbf{u}(t)$. Generally, there are two kinds of thrust which can be used for the spacecraft orbital transfer: continuous thrust and impulsive thrust. However, it is difficult to directly obtain the continuous thrust in practice. The orbital transfer strategies based on continuous thrust are always realized by multiple impulse with short constant interval or multiple impulse with constant velocity increment, which are actually two special cases of impulsive thrust.