ARTICLE IN PRESS
AESCTE:3025
Aerospace Science and Technology ••• (••••) •••–•••
Contents lists available at ScienceDirect
Aerospace Science and Technology
www.elsevier.com/locate/aescte

A new trigonometric zigzag theory for static analysis of laminated composite and sandwich plates

Rosalin Sahoo, B.N. Singh *

Department of Aerospace Engineering, Indian Institute of Technology Kharagpur, Kharagpur 721302, India

A R T I C L E I N F O

Article history:
Received 18 November 2013
Received in revised form 29 January 2014
Accepted 3 March 2014
Available online xxxx

Keywords:
Zigzag theory
Inter-laminar continuity
Laminated composite
Sandwich plate
Finite element method

A B S T R A C T

In this work, a new trigonometric zigzag theory is proposed for the static analysis of laminated composite and sandwich plates. This theory considers shear strain shape function assuming the non-linear distribution of in-plane displacement across the thickness. It satisfies the shear-stress-free boundary conditions at top and bottom surfaces of the plate as well as the continuity of transverse shear stress at the layer interfaces obviating the need of an artificial shear correction factor. An efficient displacement based C^0 finite element model is employed for the accurate assessment of the static behavior of laminated composite and sandwich plates. Some numerical examples covering various features such as different material properties, loading and boundary conditions of cross-ply composite and sandwich composite plates are solved. Efficiency and applicability of the present model is ascertained by validating the evaluated results not only with three-dimensional elasticity solutions but also with the available published results based on other shear deformation theories.

1. Introduction

The impact of composite materials is very substantial for the structural applications where high strength to weight and high stiffness to weight ratios are required specifically in aerospace industry as aircraft and spacecraft are typical weight-sensitive structures. Composite materials can also be tailored to efficiently meet design requirements of strength, stiffness, weight, corrosion resistance, fatigue life, thermal insulation, and other parameters all in various directions which enhances its applications in other industries i.e., civil, naval, architecture etc. The superiority of composite materials is very much compelling to make the research and development to be carried out across broad fonts. Laminated composites are weak in shear because of their low shear modulus correspond to the extensional rigidity which is one of its significant feature. The effect of transverse shear deformation is more significant in case of the laminated sandwich structure due to the wide variation in material properties of core and face sheets. A clear understanding on shear deformation is required to achieve the full range of capabilities on the exemplary performance of laminated composite/sandwich structures. Hence, the development of an accurate model to represent the behavior of composites has drawn an appreciable amount of attention in recent past.

The three-dimensional (3D) exact solutions are required for the prediction of accurate response of the composite structures [67,43,44]. However, such 3D solutions are restricted to simple boundary conditions, loading and geometries and as 3D finite element problems are computationally too expensive and intractable, two-dimensional (2D) laminated plate models have drawn considerable amount of attraction. The widely used displacement-based 2D plate theories can be classified into Equivalent single layer theory (ESLT), Layer wise theory (LWT) and Zigzag theory (ZZT). The classical lamination plate theory (CLT), based on Kirchoff's assumptions, predicts erroneous results due to the non-consideration of transverse shear deformation. The first-order shear deformation theory (FSDT) [55,72] assumes a constant shear deformation through the entire thickness of the laminate, that makes the model insignificant as shear deformation varies with loading, stacking sequence and boundary conditions according to Pai [45]. To overcome these drawbacks, several higher order shear deformation theories (HSDT) based on Taylor's series expansion of higher order terms [34,32,53] and expressed in terms of shear strain function [69,66,5,26], are developed which completely obviates the requirement of shear correction factor. A brief review on the shear deformation theories [56,42,54,36] ensures the continuous development of laminated theories. All the above theories fall under ESLT where the whole laminate is modeled as an equivalent single anisotropic layer and the deformation of the plate is represented in the form of unknown variables of middle plane of the plate. However, ESLT is inappropriate to predict the nature of composite

* Corresponding author. Fax: +91 3222 255303.
E-mail address: bnsingh@aero.iitkgp.ernet.in (B.N. Singh).

http://dx.doi.org/10.1016/j.ast.2014.03.001
1270-9638/© 2014 Elsevier Masson SAS. All rights reserved.

Nomenclature

Notations

a, b length and width of the rectangular plate
h total thickness of the plate
$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ Cartesian co-ordinate system
E_{11}, E_{22}, E_{33} Young's moduli of elasticity
G_{12}, G_{23}, G_{13} Shear moduli of elasticity of individual layer
$\upsilon_{12}, \upsilon_{23}, \upsilon_{13}$ Poisson's ratios
α angle of fiber orientation with respect to the $\boldsymbol{x}$-axis
$\overline{U}_1, \overline{U}_2, \overline{U}_3$ displacement components in $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$ directions
u, v, w mid-plane displacements
n_u, n_l number of upper and lower layers respectively
$\alpha^i_{xu}, \alpha^i_{yu}$ change of slopes at upper ith interface between ith and $(i+1)$th layer
$\alpha^j_{xl}, \alpha^j_{yl}$ change of slopes at lower jth interface between jth and $(j+1)$th layer
$H(z - z^u_i), H(-z + z^l_j)$ Heaviside step functions for upper and lower layers respectively
$\sigma_{xx}, \sigma_{yy}, \tau_{xy}, \tau_{xz}, \tau_{yz}$ stresses at a point

plate accurately as the transverse shear strain components are continuous across layer interfaces due to different values of shear rigidity at the adjacent layers and practically it is just the reverse.

In order to counteract the above disparity, Lu and Liu [35] and Robbins and Reddy [57] approached layer wise theory (LWT) where an unique displacement field is considered in each layer and displacement continuity is maintained across the layers taking unknowns at each layer interface. The trigonometric sinus function [69] was used in LWT by Arya et al. [2], Shimpi and Ghugal [64], Shimpi and Aynapure [63] for the analysis of composite beams. The trigonometric LWT was modeled by Mantari [37,39] for the analysis of composite plates. The finite element approach of Vidal and Polit [70], Gaudenzi et al. [25], Pai and Palazotto [46] and the work of Roque et al. [59] using mesh less method is quite appreciable. Ferreira et al. [24] analyzed behavior of the composite structures using multiquadrics discretization. Plagianakos and Saravanos [50] studied the composite and sandwich structure using higher order LWT. Toledano and Murakami [68] and Carrera [7] proposed the mixed layer wise models which predict inter-laminar stresses and displacements directly. The representation of inter laminar stresses by LWT is quite accurate at the cost of expensive computational effort since the number of unknown field variables are layer dependent.

The drawbacks of LWT are compensated by Di Sciuva [19] and Liu et al. [33] with the development of zigzag (ZZ) theory that satisfy the transverse shear stress continuity at the layer interfaces making the number of variables layer independent. Further, the stress free boundary conditions at top and bottom of the plates are also satisfied. Bhasker and Varadan [6] and Lee et al. [31] superimposed the piecewise linearly varying field on an overall higher order variation of in-plane displacements. Approach of Cho and Parmerter [12,13] was to introduce Heaviside step function to counteract the zigzag concept in a refined higher order shear deformation theory. Di Sciuva [20,21] modified his work [19] including piecewise cubic through thickness variation of the in-plane displacement which satisfies traction-free boundary conditions at top and bottom surfaces and inter-laminar continuity at layer interfaces as well. An inter-laminar shear stress continuous plate theory by Chakrabarti and Sheikh [10] and a third order ZZ theory by Kapuria and Kulkarni [28] were developed for the analysis of composite and sandwich plate. The improved higher order ZZ theory of Pandit et al. [47,48] and Chalak et al. [11] assumed the transverse displacement to vary quadratically over the core and remain constant over the upper and lower face sheets to capture core compressibility effect. Sahoo and Singh [60,61] developed a new ZZ theory combining the shear strain shape function with ZZ concept. A significant review on zigzag theories was carried out by Carrera [8]. Carrera [9], Demasi [17,18,16], Rodrigues et al. [58] and Neves et al. [41] employed Murakami's zigzag function [40] to explain the structural kinematics of the laminated plates.

Notably, analytical solutions are restricted up to certain boundaries of laminated plate made of specific lamination schemes. A number of numerical approaches are available to overcome these disparities in analyzing the structures. The finite element method (FEM) is proved to be the most powerful and versatile numerical tool for structural analysis due to its accuracy and generality. Di Sciuva [22] considered a four-noded rectangular element with 40 degrees of freedom (DOF) and a three-node conforming triangular element with 30 DOF [23] for the finite element analysis of composite plate based on the cubic zigzag theory. Cho and Parmerter [14] used a three-node triangular element whereas Chakrabarti and Sheikh [10] have developed a six node non-conforming triangular element having 42 DOF for the FE analysis of composite structures. The FE analysis based on first-order ZZ theory was carried out for the composite structures [3,4]. Kapuria and Kulkarni [28] developed a discrete Kirchoff quadrilateral element based on third order ZZ theory. Pandit et al. [47] and Chalak et al. [11] used a nine node rectangular element with 99 DOF whereas Singh et al. [65] and Sahoo and Singh [60,61] used an eight node quadrilateral element with 56 variables for the analysis of composite and sandwich structures.

Considering all these aspects of the literature review, it may be perceived that, the chosen displacement field of zigzag theory that comprises of the increased higher order polynomial terms, predict stresses accurately at layer inter-faces, at the cost of high computational efforts. Therefore, to analyze the static behavior of structure accurately with less computational effort, minimum number of field variables should be preferred with the satisfaction of traction free boundary condition and inter-laminar continuity. Henceforth, the development of a new displacement field is required which satisfies not only the tangential-stress-free boundary conditions at upper and lower surfaces of the plate, but also inter-laminar continuity condition among each layer considering less number of variables. According to authors' knowledge, it may be observed that no displacement field has been recommended so far in open literature where trigonometric function such as secant has been superimposed with zigzag theories and there are no FEM results published yet introducing the trigonometric functions as shear strain shape functions incorporating with zigzag concept.

Therefore, keeping all these viewpoints in mind, an attempt has been made in the present study to develop a new trigonometric zigzag theory to analyze the static structural behavior of laminated composite and sandwich plate which may be designated as Zigzag Theory based upon Secant Function (ZZTSF). In this theory, the in-plane displacement fields are assumed to be the combination of a linear zigzag function with different slopes at each layer and a trigonometric shear strain shape function. The transverse displacement is preferred to be constant through the thickness. A trigonometric function particularly secant function is considered as the shear strain shape function, which assumes non-linear distribution of transverse shear stresses. Further, the proposed theory fulfills the traction free boundary conditions at laminate surfaces,

Please cite this article in press as: R. Sahoo, B.N. Singh, A new trigonometric zigzag theory for static analysis of laminated composite and sandwich plates, Aerospace Science and Technology (2014), http://dx.doi.org/10.1016/j.ast.2014.03.001