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Distributed UAV formation control using differential game approach

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ABSTRACT

This paper considers a formation control problem for a multiple-UAV (unmanned aerial vehicle) system where each UAV is able to exchange information with other UAVs according to a fixed information graph. In this paper, each UAV tries to minimize its own performance index which is chosen independently based on its local information. Because of the UAVs' different objectives, the formation control problem is formulated and solved as a differential game problem. Realizing the incapability of the classical Nash strategy approach in dealing with the distributed information, we propose a novel open-loop Nash strategy design approach for each UAV to implement in a fully distributed manner through estimating its terminal state. An illustrative example of a five-UAV formation control problem is solved under different scenarios.

1. Introduction

A multiple-UAV (unmanned aerial vehicle) system is often characterized by an environment with physical constraints such that each UAV can only exchange information with neighboring ones. Because of this constraint, the control design for each UAV utilizing only the information available to it becomes a challenge. An important application of this system is the multiple-aircraft (including multi-UAV) formation control problem which is to design control inputs such that a prescribed formation is formed among the aircrafts. In recent years, a variety of results on aircraft formation control have emerged. Some of them are reviewed as follows. In [21], the decentralized overlapping control was design to control a group of interconnected UAVs, where a feedback controller was designed in the expanded space for each UAV and then converted back to the original space. In [7], the aerodynamics coupling effects of the formation flying system was studied and the trajectory tracking control and formation keeping control were combined and designed using linear quadratic regulator approach. In [10], the high-level formation control problem of organic air vehicles (OAVs) was considered using receding horizon control approach. In [22], a unified optimal control approach including formation control, trajectory tracking, and obstacle avoidance was proposed for multiple-UAV coordination. In [23], the fuel optimization of formation initialization problem for spacecraft was considered and the optimization was convexified and solved as a semidefinite program. In [11], the attitude synchronization problem of the spacecraft was considered and the decentralized control algorithm was developed based on nonlinear cooperative control theory. A comprehensive review in larger scope on multi-agent control systems including recent progress on aircraft formation control can be found in [6]. Most of the recent results on formation control problem utilize tools such as cooperative control theory [17,16], optimal control theory [5,2], receding horizon control [15,4], etc., and all the aircrafts are usually assumed to pursue a common goal of minimizing the total formation errors and velocity differences among them. However, it is of practical interest to have a more general setting where individual aircrafts can have their own objectives. For example, one aircraft's objectives might be chosen based on its locally measured formation errors and velocity differences. Therefore, given the aircrafts' different objectives, the formation control problem indeed becomes a differential game problem [9]. However, only a few research works have been done in this area. In [8], the formation control problem was formulated as a noncooperative differential game and the receding horizon Nash equilibrium was solved. In [18], the consensus problem as a special case of formation control problem was formulated as a cooperative differential game and the Nash bargain solution among the Pareto-efficient solutions was found using linear matrix inequality (LMI) approach. In this paper, based on the previous results, we consider the distributed Nash strategy design approach for multiple-UAV formation control problem. We will propose a novel approach that enables each UAV to implement its Nash strategy only based on the information available to it only.

The rest of the paper is organized as follows: The problem is formulated in Section 2. We derive the classical open-loop Nash equilibrium in Section 3. The distributed Nash strategy design approach is provided in Section 4. An illustrative example of

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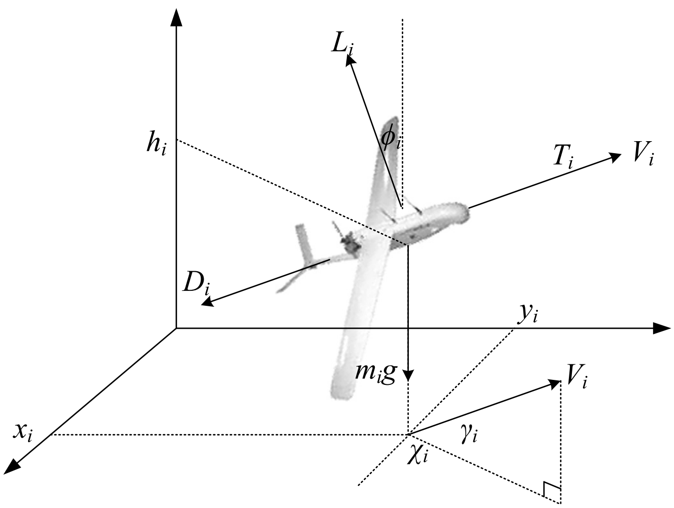

Fig. 1. UAV model.

a five-UAV formation control problem is solved in Section 5. The paper is concluded in Section 6.

2. Problem formulation

2.1. UAV model

Since there exist various UAVs that are designed to complete different real life tasks, it is impossible to have one universal mathematical dynamic model to describe all the UAVs. This paper only focuses on the high-level formation control design among a group of UAVs and hence will adopt a representative UAV model which has been commonly used in many literatures [13,16,22]. We consider a system of N UAVs with the following point-mass model [13] as shown in Fig. 1:

$$\dot{x}_i = V_i \cos\gamma_i \cos\chi_i, \tag{1}$$

$$\dot{y}_i = V_i \cos\gamma_i \sin\chi_i \tag{2}$$

$$\dot{h}_i = V \sin\gamma_i \tag{3}$$

$$\dot{V}_i = \frac{T_i - D_i}{m_i} - g\sin\gamma_i \tag{4}$$

$$\dot{\gamma}_i = \frac{L\cos\Phi_i - m_i g\cos\gamma_i}{m_i V_i} \tag{5}$$

$$\dot{\chi}_i = \frac{L_i \sin\Phi_i}{m_i V_i \cos\gamma_i} \tag{6}$$

for $i = 1, \dots, N$, where x_i is the down-range displacement, y_i is the cross-range displacement, h_i is the altitude, V_i is the ground speed which is assumed to be equal to the airspeed in this paper, γ_i is the flight path angle, χ_i is the heading angle, T_i is the engine thrust, D_i is the drag, m_i is the UAV mass, g is the acceleration due to gravity, L_i is the lift, and Φ_i is the banking angle. The three control inputs of UAV i is the banking angle Φ_i, lift L_i, and engine thrust T_i.

It is shown in [13] that the highly nonlinear UAV model in (2.1) can be pre-linearized using feedback linearization to be

$$\ddot{x}_i = u_{xi}, \qquad \ddot{y}_i = u_{yi}, \qquad \ddot{h}_i = u_{hi} \tag{7}$$

where u_{xi}, u_{yi}, and u_{hi} are the virtual acceleration control inputs. These virtual control inputs and the real control inputs are related through the following equations

$$\Phi_i = \tan^{-1}\left(\frac{u_{yi}\cos\chi_i - u_{xi}\sin\chi_i}{(u_{hi}+g)\cos\gamma_i - (u_{xi}\cos\chi_i + u_{yi}\sin\chi_i)\sin\gamma_i}\right) \tag{8}$$

$$L_i = m_i \frac{(u_{hi}+g)\cos\gamma_i - (u_{xi}\cos\chi_i + u_{yi}\sin\chi_i)\sin\gamma_i}{\cos\Phi_i} \tag{9}$$

$$T_i = m_i\big[(u_{hi}+g)\sin\gamma_i + (u_{xi}\cos\chi_i + u_{yi}\sin\chi_i)\cos\gamma_i\big] + D_i \tag{10}$$

where $\tan\chi_i = \dot{y}_i/\dot{x}_i$ and $\sin\gamma_i = \dot{h}_i/V_i$. Therefore, after the virtual control inputs are designed based on the linear model (7), the real control inputs can then be obtained by substituting the virtual ones into (7). Expressing (7) in terms of state-space representation yields

$$\dot{z}_i = Az_i + Bu_i, \tag{11}$$

$$p_i = C_p z_i \tag{12}$$

$$v_i = C_v z_i \tag{13}$$

where $z_i = [p_i^T \ \ v_i]^T$ is the state vector, p_i is the position vector, v_i is the velocity vector, $u_i = [u_{xi}^T \ \ u_{yi}^T \ \ u_{hi}^T]^T$ is the virtual acceleration control vector,

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_3, \qquad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes I_3,$$

$$C_p = [1 \quad 0] \otimes I_3, \qquad C_v = [0 \quad 1] \otimes I_3,$$

$I_3 \in \mathbb{R}^{3\times 3}$ is the identity matrix, and $\otimes$ is the Kronecker product.

2.2. Information graph

Suppose that individual UAVs are able to communicate with each other in a certain pattern to achieve the desired formation. We define a time-invariant directed information graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ to describe the information exchange pattern among them. Specifically, node $v_i \in \mathcal{V}$ represents UAV i and edge $e_{ij} \in \mathcal{E}$ represents the directional information transmission from UAV j to UAV i. Several terminologies from graph theory are introduced as follows.

Definition 1. Node i is globally reachable in graph $\mathcal{G}$ if there exists a sequence of edges directed from v_i to v_j for all $j = 1, \dots, N$, $j \neq i$.

A globally reachable node is also known as a root node of a spanning tree on the graph. Based on the definition of globally reachable node, the connectivity of a graph is defined as follows.

Definition 2. Graph $\mathcal{G}$ is connected if there exists at least one globally reachable node.

In this paper, to achieve the formation requirement, the connectivity of the UAVs on the information graph must be assured. Hence, we make the following assumption:

Assumption 1. The underlying information graph among the N UAVs is connected.

A widely used mathematical tool in graph theory is the Laplacian matrix $\mathcal{L} = [\mathcal{L}_{ij}] \in \mathbb{R}^{N\times N}$ which is defined as follows:

$$\mathcal{L}_{ij} = \begin{cases} -l_{ij} & \text{if } e_{ij} \in \mathcal{E} \text{ for } j \neq i \\ 0 & \text{if } e_{ij} \notin \mathcal{E} \text{ for } j \neq i \\ -\sum_{q=1,\, q\neq i}^{N} l_{iq} & \text{if } j = i, \end{cases} \tag{14}$$

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