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Fundamental theories of aerodynamic force in viscous and compressible complex flows

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ABSTRACT

In recent decades the major tools of aerodynamics have been computational fluid dynamics (CFD) and experimental fluid dynamics (EFD) but not theoretical methods. This abnormal phenomenon is often attributed to the lack of theories for viscous complex flows we are now facing with in classic aerodynamics. In this article we show that such an attribution is oversimplified. Instead, the general theoretical foundation needed for today's aerodynamics was already settled by those pioneering masters a century ago as their great legacy, from which modern fundamental force theories applicable to complex flows have grown up. They are formulated at four levels, with each level having its special ability to reveal the flow physics. These theories can well be combined with CFD and EFD for practical complex flow diagnosis and configuration design. In the article, the underlying physics of the relevant theories is always put to the first place, but some remarks on key historical events are also made. Examples are given to demonstrate the power of the theories in complex-flow diagnosis, design, and flow control, along with some new simplified theoretical models derived thereby.

1. Introduction

1.1. Theoretical aerodynamics as the bridge of CFD data and forces

Modern aerodynamics is facing various complex flows involving boundary-layer separation, free shear layers, vortices and shocks, as well as the interactions of these structures, for which the flow data can in no way be obtained by analytical methods. Thanks to the rapid development of computational fluid dynamics (CFD) and experimental fluid dynamics (EFD), the desired data can now be acquired by these powerful means, and therefore the aerodynamic forces could be calculated by body-surface stress integral that we call standard formula. However, the stresses on the body-surface cannot be measured easily and accurately, especially when the body-shape or the flow itself is very complicated. What is more important, obtaining the flow data is only the first and necessary step toward fully understanding the flow. The *quantitative* relation between the complex flow structures and the aerodynamic forces can never automatically surface from these data themselves nor from the standard formula. Without this relation one would be blocked at the phenomenological and empirical level, a rational CFD/EFD-based vehicle design would be impossible, and the huge flow data would be mostly wasted. To find the desired quantitative relation a firm bridge is necessary, which can be nothing else but theoretical aerodynamics to be addressed in this article, since only theories can extract physical rules from computed or measured flow data and thereby have predictive power.

Specifically, this article will concentrate on the *fundamental theories* of aerodynamic forces for complex flows. These theories are required to be as generally effective as possible, independent of specific initial-boundary conditions. They start from the first principles which, by using governing equations and some transformations, lead to certain formulations. Their product is various integral-form formulas with integrands consisting of unknown flow variables, linearly or nonlinearly. Thus, these theories have only partial predictive power in the sense that, for finding the forces, they tell what flow variables in particular domain one should focus on. Although under the same conditions these theories finally yield the same values of the force, a fundamental theory will be considered better than others if it involves fewer unknown variables in more localized integration domains (a property to be said “neater”), and it can reveal more clearly the key physics underlying the force.

An aerodynamic force theory would have fully predictive power only if the relevant flow field has been solved under the prescribed initial-

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boundary conditions, so that in its force formula there is no unknown flow variable but just the body geometry and flow parameters. In classic (pre-CFD) era such a full predictive ability was achieved by *specific simplified theories* derived from a couple of fundamental theories, where the approximate solutions of certain simple flows can be analytically obtained. However, when the flow becomes complex, a fully predictive ability can only come from a proper combination of fundamental theories and CFD/EFD governed by the same set of Navier-Stokes (NS) equations. A direct role of such combination is *complex-flow diagnosis*, which could be developed to more advanced methods of optimal aerodynamic design and flow control.

To understand how the desired theoretical bridge could be constructed on a broader background, in what follows we briefly review the evolution history of theoretical aerodynamics from classic to modern eras.

1.2. The rise and fading of classic theoretical aerodynamics

To a very large extent, the history of classic theoretical aerodynamics is a history of developing brilliant simplified force theories. Among these are the well-known formulas of lift coefficient $C_l = 2\pi\alpha$ for two-dimensional (2D) thin-airfoil in a steady flow with small angle of attack α [1], of induced-drag coefficient $C_D = C_L^2/(\pi\Lambda)$ for three-dimensional (3D) elliptic wing of aspect ratio Λ [2], the lift and moment of oscillatory airfoil [3] and of general unsteady airfoil [4]. They have fully predictive ability and were all based on a common assumption: the body motion causes only small disturbances to the uniform fluid. Therefore, the flow remains fully attached (known as *simple flow*) where, at sufficiently large Reynolds number, the global flow is governed by the velocity-potential equation under simplified boundary conditions. In this case, the matured and rich mathematic techniques can be elegantly employed, and the approximate solutions can be obtained in closed or series form. The viscous vortical-flow effects are confined to very thin boundary layers and wakes that can be accounted for afterwards. This approach, by neglecting minor processes for specific flow problems and using simplified theoretical models, is called *"theoretical modeling method"*, which is named and practiced by B. G. Tong and his collaborators [5–7] as an inheritance and specification of the concept of Tsien [8] on *technical science* (or *engineering science*).

The theoretical-modeling method continued as one entered high-speed regime; thus from their very beginning, simplified high-speed theories were characterized by small-perturbation potential flows over thin or slender bodies in a Mach-number range from subsonic to hypersonic, associated with their respective similarity rules. All these theories successfully ensured the rapid development of aeronautical engineering. Since this strategy exhibited satisfactorily predictive ability within its application range, von Kármán [9, p. 2] proudly stated: *"Mathematical theories from the happy hunting grounds of pure mathematicians were found suitable to describe the airflow produced by aircraft with such excellent accuracy that they could be applied directly to airplane design."*

The turning point from simple flow to complex flow is the occurrence of boundary-layer separation, where the mathematical theories mentioned by von Kármán could no longer help and the task of obtaining flow data was shifted to the hand of CFD and EFD. On the other hand, being of little help for reading off the precise key physics from measured or calculated data and unable to guide the advancement of CFD and EFD, classic theoretical aerodynamics has lost impetus to move on. As a result, the progress in CFD (and EFD) has not propelled our understanding of aerodynamics much further forward, although most of the physical insight gained has arisen from classic aerodynamics [10]. Indeed, according to M. Gad-el-Hak (2016, private communication), the priority order of theoretical and experimental aerodynamics in the 1950s was theory-experiment, and became theory-experiment-computation in the beginning of the modern era; but as the great capability of CFD was quickly exhibited, the order changed to experiment-computation in the 2000s. Then since about 2010 it changed again to computation-experiment. Theoretical aerodynamics has faded away in the 21st century.

1.3. Development and challenge of modern theoretical aerodynamics

Any branch of science cannot develop healthily without theory. The fading of theoretical aerodynamics in recent decades is an abnormal phenomenon that cannot last long. This phenomenon is associated with a widely spread viewpoint that classic aerodynamics consists of merely those simplified force formulas that are indeed useless for complex flows. But this viewpoint ignores the very existence of *classic fundamental theories* that have general applicability and are by no means out of date. Here, special honor should be given to the famous Kutta-Joukowski (KJ) theory to be addressed later in § 3, the earliest and one of the most universal fundamental theories; Prandtl's general *vortex-force theory* [2] in § 4, the 3D extension of the KJ theory that can be smoothly extended to viscous and unsteady flow; and the general *impulse theory* pioneered by Burgers [11] in § 5, a fundamental theory applicable to any unsteady flow over an arbitrarily moving and deforming body with extreme neatness.

Actually, the familiar simple-flow theories mentioned in the beginning of § 1.1 are just the linearized or quasi-linearized versions of the above fundamental theories: the thin-airfoil theory [1] was from the KJ theory; Prandtl's lifting-line theory [2] was from his own vortex-force theory in the same paper; and Kármán-Sears (KS) unsteady theory [4] was a 2D inviscid and linear approximation of the impulse theory. But in contrast to these special simple-flow theories, their "mother theories" are all generally valid. We believe that the vortex-force theory (including the KS theory as its 2D version) and impulse theory are the two brilliant pearls on the crown of classic aerodynamics. Thus, by as early as 1920s, the general basis of viscous aerodynamics for incompressible flows, both steady and unsteady, was already laid down.

Then, after a silence for about half century, new formulations of viscous aerodynamic force appeared since 1980s, and by now the desired modern theories in low-speed regime have been quite rich. In contrast, since the viscosity was dropped at its beginning, classic high-speed force theory never led to any fundamental one with sufficient generality other than a fully nonlinear compressible Bernoulli equation. But the pioneering work of Chang et al. [12] showed that one can smoothly extend the above classic fundamental incompressible theories to viscous compressible flow without principal difficulty [13,14]. Therefore, thanks to the classic vortex-force theory and impulse theory, a set of fundamental force theories for complex viscous and compressible flow, steady or unsteady, have now grown to an active research field. These theories form a sound foundation or skeleton of modern theoretical aerodynamics and are the main topics of the present article.

In addition to be combined with exact fundamental theories, the CFD/EFD can also be combined with certain specific simplified theories that have more predictive ability than general theories. The continuous need for theoretical-modeling method comes from the fact that, because facing complex flows with different physical processes coupled together (see § 2.2), it is of crucial importance to pinpoint the key process that dominates the performance of aerodynamic force.

On the other hand, although it is no longer hopeful to obtain approximate global analytical solutions as for simple flow, aerodynamic forces are strongly influenced by a number of key *local* physical processes, of which the mechanisms have to be clarified as much as possible by relevant localized specifications of the NS equations. The most influential and elegant example is the triple-deck theory [15–17], which clarifies quantitatively the formation of free shear layer from separation of boundary layer at smooth surface, corner, or trailing edge of the wall, that is the very physics behind the transition from simple flow to complex flow. Of its further developments and applications is the theory on turbulent shock-boundary-layer interactions [18]. Other crucial local processes still under study include, e.g. (these subjects have been well documented, and the citations here are just a few recent advances