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Towards a standard design model for quad-rotors: A review of current models, their accuracy and a novel simplified model

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A R T I C L E I N F O

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A B S T R A C T

Applications based on *quad-rotor-vehicles* (*QRV*) are becoming increasingly wide-spread. Many of these applications require accurate mathematical representations for control design, simulation and estimation. However, there is no consensus on a standardized model for these purposes. In this article a review of the most common elements included in *QRV* models reported in the literature is presented. This survey shows that some elements are recurrent for typical non-aerobatic *QRV* applications; in particular, for control design and high-performance simulation. By synthesising the common features of the reviewed models a *standard generic model SGM* is proposed. The *SGM* is cast as a typical state-space model without memory-less transformations, a structure which is useful for simulation and controller design. The survey also shows that many *QRV* applications use simplified representations, which may be considered simplifications of the *SGM* here proposed. In order to assess the effectiveness of the simplified models, a comprehensive comparison based on digital simulations is presented. With this comparison, it is possible to determine the accuracy of each model under particular operating ranges. Such information is useful for the selection of a model according to a particular application. In addition to the models found in the literature, in this article a novel simplified model is derived. The main characteristics of this model are that its inner dynamics are linear, it has low complexity and it has a high level of accuracy in all the studied operating ranges, a characteristic found only in more complex representations. To complement the article the main elements of the *SGM* are evaluated with the aid of experimental data and the computational complexity of all surveyed models is briefly analysed. Finally, the article presents a discussion on how the structural characteristics of the models are useful to suggest particular *QRV* control structures.

1. Introduction

The use of *unmanned aerial vehicles* (*UAV*) is currently becoming part of the normal operating procedures for many public and private organizations. In particular, security, surveillance and research applications are widespread [1]. The growth of these applications has brought technological developments in several areas such as sensors, programming techniques, digital processing units, energy storage, etc [2].

*UAV*s can be classified, according to their physical configuration, as fixed or rotatory wing vehicles. The emergence of new applications for *UAV*s has also stimulated the design of innovative configurations. For instance, an unconventional vertical take-off and landing configuration is suggested in Ref. [3]. While in Refs. [4,5], a ducted fan micro *UAV* and cyclocopter are reported. Other unconventional configurations such as ducted fan micro *UAV* and cyclocopters are reported in Refs. [4,5].

One of the most recurrent configurations in several applications is the *quad-rotor-vehicle* (*QRV*). These vehicles have the characteristic of being easy and cheap to construct. In addition, it has been shown that in practice these vehicles can be stabilised with fairly simple linear controllers. In fact, several working *QRV* applications use traditional linear PI controllers with excellent experimental results [6]. Nevertheless, the design of many of these controllers remains mostly heuristic, mainly due to the nature of the existing *QRV* dynamical models. Generally, these complex models invite to consider more sophisticated control strategies in order to achieve asymptotic stability. For instance, in Refs. [7,8], backstepping based control has been successfully used. Other approaches

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such as adaptive and robust backstepping are reported in Ref. [9]. Feedback linearisation and sliding mode control are reported in Refs. [10,11]. Additional non-linear control schemes for *QRV*s can be found in Ref. [2].

Notwithstanding the numerous reports on successful missions carried out using these vehicles, there is still a lack of a standard *QRV* model. In fact, many of the reports start with a modelling procedure. The lack of a standard design model is remarkable considering that other aerial vehicles, such as helicopters or fixed-wing aeroplanes have fairly standardized models. From the point of view of model-based control design, the provision of a valid theoretical model consists in one of the most important steps, if not the most important.

Another approach for *QRV* control is to use a specialized model for a particular vehicle dynamic or operating condition. For instance, the following reports solve a particular *QRV* control problem using different models and control strategies:

- For altitude control: H_∞ linear controllers [12], robust pole placement [13,14], feedback linearisation [15–17] and other nonlinear methods [18,19].
- For attitude control: classic linear controllers [20,21], *LQR* [19,21], H_∞ linear control [12], feedback linearisation [15,16], double lead compensator [21] and other nonlinear methods [14,22].
- For velocity control: H_∞ linear controllers [12] and robust pole placement [23].

The heterogeneity of the design models used in the applications above-mentioned is notable. This hinders the study of *QRV* flight dynamics in comparison with classical configurations. For instance, the derivation of key dynamical behaviours such as *phugoid* or *duch-roll* equivalents in fixed wing aeroplanes or helicopters [24] is not possible for *QRV*s due to the lack of a proper standardized model.

This article presents a comprehensive review of existing *QRV* models used mainly for control design and real-time simulation. Through this review it is shown that most of the reported models can in fact be considered as a simplification of a more complex representation. This model can be considered as a *standard generic model SGM* for *QRV* control design and simulation. On the other hand, motivated by the complexity of the *SGM* a study leading to valid simplifications is also included.

Although many of the models reported in this article have been already validated individually numerically and experimentally, a comparison among these models under the same conditions is missing in current literature. Therefore, the accuracy of all the simplified models found in the literature is revised through a comprehensive set of numerical evaluations under a wide range of conditions. As expected, not all the simplified models are valid for all the operating conditions. This information, which is non-existent in the current literature, is crucial since *many of the models are shown not to be appropriate for typical QRV applications*, such as trajectory tracking with yaw angle movement. In this context, it is reasonable to wonder if the use of inadequate models is one of the main factors leading to a lengthy trial-and-error controller tuning.

In addition to the review and evaluation of existing models, a novel simplified model is derived by analysing the input-output characteristics of the system. The new model is easy to linearise via feedback linearisation and yields a good compromise between simplicity and accuracy. Moreover, the proposed model is shown to be useful for the design of simple controllers using classical frequency analysis tools. In particular, a controller for position and yaw tracking is designed using the proposed simplified model. The resulting controller yields an excellent performance even with the most comprehensive model in a very wide range of operation.

The article is complemented with an experimental assessment of the *GSM* and a brief discussion on the computational complexity of the models. This aspect becomes important for collaborative flight formation of *QRV* s, which require the synchronized navigation of multiple vehicles at the same time [25,26]. Finally, Tables 1 and 2 present the main acronyms and variables used along the article.

Table 1
Acronyms.

Acronym	Definition
QRV	*Quad-Rotor Vehicle*
SGM	*Standard Generic Model*
QSMi	*Quadrotor Simplified Model i*
QL	*Quasi Linear* quadrotor model
QL_o	*Quasi Linear* quadrotor orientation model
TNL	*Complete nonlinear model*
MIE	*Mean absolute Input Error*
MOE	*Mean absolute Output Error*
TME	*Total Mean Error, MIE + MOE*
TMER	*Total Mean Error Ratio, TME* relative to range
CoG	*Center of Gravity*

Table 2
Definition of variables.

Variable	Description
m	Vehicle mass
J	Inertial moments matrix
F_b, M_b	External forces and moments vectors in body reference frame
V_b, V_e	Translational velocity in body and inertial reference frames
ω_b	Angular velocity in body reference frame
I_α	Inertial mass along x and y axes
I_z	Inertial mass along z axis
$\theta - \phi - \psi$	Euler angles: pitch-roll-yaw
R	Rotation matrix with sequence $\psi - \theta - \phi$
R_ψ, R_θ, R_ϕ,	Rotation matrices along ψ,θ,ϕ
c_θ, s_θ, t_θ	Short for $\cos(\theta), \sin(\theta), \tan(\theta)$
ω_i	Angular velocity of propeller i
$\omega_\Gamma = 0$	Total angular velocity of propellers producing gyroscopic force
M_Γ	Gyroscopic moment due to propellers
k_p	Propeller-motor thrust coefficient
k_m	Propeller-motor moment coefficient
V_i	Input voltage to motor i
ℓ	Propeller moment arm length
F_z	Propulsion force in z axis
T_p, T_q, T_r	Propulsion moments around x,y,z axes
$\overline{x}$	Approximation to x
U_x, U_y, U_z	Virtual inputs to variables x,y,z
θ_r, ϕ_r	Virtual pitch and roll angles
x_0	Equilibrium point of variable x

2. Searching for a standard generic model

The bibliographical review regarding *QRV* applications reveals that there are three fundamental approaches for obtaining the mathematical models from fundamental principles:

- Newton-Euler: This is the most common approach for aerial vehicle modelling. It has the advantage of using Euler angles and Newton laws, which are easy to understand and to relate with the actual application. The simplification of these models yields mathematical structures that are easy to manipulate. In addition, the majority of the simplified models found in the current literature can be considered simplifications of the Newton-Euler structure. The disadvantage of this method is that Euler angles present a singularity when the second Euler angle reaches ±90°. This may limit the use of these representations for special applications, such as aerobatic manoeuvres.
- Euler-Lagrange: This approach is based on energy modelling principles. The resulting model is dynamically equivalent to those derived using the Newton-Euler approach. However, the mathematical structure that supports this approach is different, some mathematical manipulation is required to arrive to the equivalent Newton-Euler structure. Since this method is also based on Euler angles it also suffers from the same singularity problem as the Newton-Euler model. These models are normally either simplified to an equivalent Newton-