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# Sensitivity analysis for decision-making using the MORE method— A Pareto approach

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ABSTRACT

Integrated Assessment Modelling (IAM) incorporates knowledge from different disciplines to provide an overarching assessment of the impact of different management decisions. The complex nature of these models, which often include non-linearities and feedback loops, requires special attention for sensitivity analysis. This is especially true when the models are used to form the basis of management decisions, where it is important to assess how sensitive the decisions being made are to changes in model parameters. This research proposes an extension to the Management Option Rank Equivalence (MORE) method of sensitivity analysis; a new method of sensitivity analysis developed specifically for use in IAM and decision-making. The extension proposes using a multi-objective Pareto optimal search to locate minimum combined parameter changes that result in a change in the preferred management option. It is demonstrated through a case study of the Namoi River, where results show that the extension to MORE is able to provide sensitivity information for individual parameters that takes into account simultaneous variations in all parameters. Furthermore, the increased sensitivities to individual parameters that are discovered when joint parameter variation is taken into account shows the importance of ensuring that any sensitivity analysis accounts for these changes.

## 1. Introduction

Integrated Assessment Modelling (IAM) incorporates knowledge from different disciplines to provide an overarching assessment of the impact of different management decisions. Such modelling methods generally require the specification of values for numerous parameters from various sources, many not known with certainty. Rapid increases in model size and complexity, particularly in the case of integrated models for decision-making, pose new challenges for effective sensitivity analysis. As IAM methods are increasingly being used to inform environmental management decisions, it is important that there are sensitivity analysis methods, which cater to the challenges posed by these models. The large and varied amount of data required for IAM means that frequently data are incomplete and model inputs are not known with certainty. Furthermore, the likely presence of feedback loops, non-linearities and non-monotonicity in IAM models compounds the uncertainty in the model outputs. Consequently, and because models do not always behave intuitively, sensitivity analysis is an important stage of model development. In the case of models used for decision-making, sensitivity analysis is important to help understand how sensitive the decisions are to changes in the values of model parameters and inputs. For simplicity, both model inputs and parameters will be referred to as parameters throughout this article.

The Management Option Rank-Equivalence (MORE) method is a new, rank-equivalence method of sensitivity analysis [1] developed specifically for sensitivity analysis of models being used for decision-making. Saltelli et al. [2] identify that sensitivity analysis should focus on the question at hand, rather than focusing solely on the model output, and resultantly indicates the creation of settings for sensitivity analysis. The setting which the MORE method is designed to address is similar to the Factors Mapping (FM) setting described by [2], which categorizes the model output $Y$ into two groups, and asks the question "which factor is most responsible for producing realizations of $Y$ in the region of interest?"

The MORE method investigates the sensitivity of the management decision guided by the model, to changes in the model parameters. The method operates on the premise that IAM used in decision-making facilitates the ranking of potential management solutions based on their efficacy of solving the particular management problem, in order to determine the most effective management solution. In a decision-making context, it is important that the solution is robust i.e. that management option rankings will not alter with small changes in model parameters. In this setting, the question that is asked is, given the model output $Y = f(\mathbf{x},\mathbf{z})$, where $\mathbf{x}$ is the parameter vector, and $\mathbf{z}$ is the

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management option, "what combined change in model parameters is required to alter the rank of the potential management solutions?"

The MORE method of sensitivity analysis incorporates numerical optimization techniques to find the minimum and maximum combined changes in parameters that cause a change in the ranking of two management options, thus altering the preferred decision. The set of parameter vector values, for which the two management options are equally ranked, is called the rank-equivalence boundary (REB), as it separates the set of parameter values where the management decision would alter, from those where it would not. The sensitivity of a decision to model parameters can be measured through the change required in model parameters to alter the decision. Where only a small combined parameter change is required the decision can be considered to be sensitive to the model parameters, whereas if large changes are required to alter the decision, it can be considered robust. Thus, by identifying the rank-equivalence boundary, the sensitivity of the decision to model parameters can be determined as the distance in parameter space between the calibrated model parameters and the REB. To overcome the difficulty of characterizing the entire REB, the MORE method constructs two artificial boundaries based on the minimum and maximum combined change in parameters to reach the rank-equivalence boundary, providing a decision maker with information about the robustness of management solutions given different parameter vector locations, as well as characterizing the sensitivity variation in different parameter directions. Similar methods have been used to assess uncertainty due to weighting of objectives in multi-criteria decision-making [3–5]. While these methods take a similar approach to the MORE method, through determining minimum changes in parameters to reach a decision threshold, they only assess the impacts of output criteria weights, rather than the model parameters, and as such tend to be focused on constraints that are linearly additive and well defined, whereas the REB used in the MORE method is likely to be highly non-linear.

Although the MORE method enables assessment of the amount of variation of the model sensitivity in all possible parameter directions, it does not provide information regarding the change in sensitivity in particular directions in parameter space, as the REB is not equidistant from the calibrated model parameters in all directions. Information about the parameter changes required to reach the REB in different directions can be obtained by examining Pareto optimal solutions [6]. These solutions are critical points on the REB, as they can be reached through small changes in parameter values that may be similar in value to the minimum combined parameter change, but occur in a different direction in parameter space. Consequently they have different ratios of parameter changes to the solution representing the minimum Euclidean distance, while still being minimum solutions.

This research proposes an extension to the MORE method, termed Pareto optimal MORE (POMORE), allowing further investigation into the variation of the sensitivity of the decision in different parameter directions. In order to locate several critical parameter combinations on the rank-equivalence boundary, a multi-objective, Pareto optimal search [7] is performed. The details of this search are outlined in Section 4.3. During the multi-objective optimization, the minimization of each individual parameter change is defined as an individual search objective and a constraint is set to restrict solutions to the rank-equivalence boundary. Unlike a weighted combination of the objectives, a Pareto optimal search returns many minimal locations on the rank-equivalence boundary, thus identifying a collection of critical points on this boundary. These critical points represent the minimum change in a single parameter, with simultaneous minimal changes in other parameters. This analysis extends beyond the single minimum change in combined parameter values, to locate other critical parameter combinations in different directions in the parameter space.

## 2. Methodology

### 2.1. MORE method

The MORE method of sensitivity analysis is for use specifically in the case where the model in question is being used to assess policy or management options. Given a model

$$y = f(\mathbf{x}, \mathbf{z}) \tag{1}$$

where $\mathbf{x}$ is a vector

$$\mathbf{x} = [x_1 x_2 \dots x_k]^T \tag{2}$$

of $k$ parameters or input factors in $\mathscr{P}$, which denotes the set of feasible parameters over which the model produces a realistic output, and $\mathbf{z}$ is the vector of management options, which are to be selected from, we can represent a realization of the parameters as $\mathbf{x}_A$ with corresponding model output $y_A(z)$. Two management options $z_1$ and $z_2$, yield model outputs $y_{A,1}$ and $y_{A,2}$.

The ranking of the management options is changed as we cross over the parameter set

$$\mathscr{B} = \{\mathbf{x} \in \mathscr{P} : f(\mathbf{x}, z_1) = f(\mathbf{x}, z_2)\} \tag{3}$$

where $\mathscr{P}$ denotes the feasible parameter set. $\mathscr{B}$ is also known as the rank-equivalence boundary, identifying the boundary in parameter space between different preferences of management options, based on the model output. The set $\mathscr{B}$ is a $(k-1)$-dimensional manifold and is the boundary of the $k$-dimensional set

$$\mathscr{B}^\star = \{\mathbf{x} \in \mathscr{P} : f(\mathbf{x}, z_1) \geqslant f(\mathbf{x}, z_2)\} \tag{4}$$

It is desirable to know the set $\mathscr{B}^\star$ as it is representative of the parameter changes that can occur while still maintaining the original management decision. However, identification of the boundary set, $\mathscr{B}$, which would allow determination of the set $\mathscr{B}^\star$, is problematic for complex models with many parameters. The MORE method [1] uses optimization techniques to locate the minimum and maximum distances from the calibrated model parameter vector to the REB, identified as $D_{\min}$ and $D_{\max}$, respectively. It then attempts to characterize the boundary set $\mathscr{B}$, and the sets that it separates, through classification of the parameter space into 3 sets based on $D_{\min}$ and $D_{\max}$. While effective in providing information about combined parameter changes, the MORE method is unable to give any information about individual parameter sensitivities. The extension to MORE SA proposed in this research attempts to identify additional critical locations on the REB, beyond $D_{\min}$ and $D_{\max}$, through multi-objective optimization, based on Pareto dominance.

### 2.2. Categorization

Several different taxonomies for categorizing sensitivity analysis methods are available in the literature. The most prevalent classifies methods based on their capabilities as being either screening methods, local methods or global methods [2], where local methods involve assessment of sensitivity at a particular location in parameter space, global methods use a variety of techniques to assess the contribution of each parameter over the entire parameter space, and screening methods provide an