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ORIGINAL ARTICLE

Flow of micropolar fluid over an off centered rotating disk with modified Darcy's law

N.A. Khan[a,*], S. Khan[a], A. Ara[b]

[a]Department of Mathematics, University of Karachi, Karachi, Sindh 75270, Pakistan
[b]Department of Computer Science, Mohammad Ali Jinnah University, Karachi, Sindh 75400, Pakistan

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Abstract The problem of the steady, incompressible, three dimensional stagnation point flow of a micropolar fluid over an off centered infinite rotating disk in a porous medium is studied in this article. Injection/suction is applied uniformly throughout the surface of porous disk. The Darcy's resistance for the micropolar fluid is also formulated. The partial differential equations are converted into the set of ordinary differential equation by utilizing the suitable transformation. The system of equations is analytically solved by the means of a non-perturbative technique, homotopy analysis method (HAM). The influence of rotational parameter, material parameter, spin gradient viscosity parameter, micro-inertia density parameter, porosity parameter and suction/injection parameter on velocity functions is presented in graphical form and discussed in detail. Verification of the solutions is made by a numerical comparison with the previous study.

1. Introduction

For a long time, the non-Newtonian fluids captivated the attention of the researchers. The great interest in non-Newtonian fluids is because of their great commercial importance. In reality, many fluids like biological fluids, slurries, shampoo, yoghurt, tomato sauce, grease, cosmetic products, paints, lubricants, polymers, custard, blood, and several other fluids do not obey the linear stress-velocity gradient relationship, which is the Newtonian fluid theory. Numerous non-Newtonian fluid models were therefore proposed to explain the complex behavior. Usually, the stress constitutive relations of such models inherit complexities, which lead to highly nonlinear

*Corresponding author. Tel.: +923333012008.
E-mail address: njbalam@yahoo.com (N.A. Khan).

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Nomenclature

x, y, z	Cartesian coordinates
u, v, w	Velocity components
r, θ	Cylindrical coordinates
p	Pressure
$\tilde{r}$	Darcy resistance
N	Micro rotation
I	body couple per unit mass
j	micro-inertia
κ	vortex viscosity
d	Distance between the axis of flow and disk axis
a	strength of the stagnation flow
M	Torque experienced by disk
c_1	material parameter
c_2	spin gradient viscosity parameter
c_3	micro-inertia density parameter
e	porosity parameter
S	Uniform injection parameter
k_1	permeability

Greek letters

$\lambda_1, \alpha_1, \beta_1$	material constant of micropolar fluid
μ	Viscosity of micropolar fluid
γ_1	spin gradient viscosity
ω	angular velocity
ρ	Density
τ	extra stress tensor
α	rotational parameter
ϕ	porosity of the porous medium

equations of motion with many terms. Most of the fluids used in the industrial applications are non-Newtonian in nature, especially, in polymer processing and chemical engineering processes etc. [1–3].

Among all the non-Newtonian fluids, the micropolar fluid has received special attention due to its additional angular momentum equation. Micropolar fluids are physically based on microstructure; these fluids may represent the fluids consisting of rigid, randomly oriented or spherical particles suspended in a viscous medium, where the deformation of fluid particles is ignored [4]. This fluid model was proposed by Eringen [5,6]. Many researchers focused on this fluid and studied it in different ways. Khan et al. [7] calculated the analytical solution of the creeping flow of two dimensional, unsteady micropolar fluid in Cartesian coordinates. Nazir and Shafique [8] investigated the numerical solution of the steady micropolar fluid flow caused by stretching cylinder, by utilizing the SOR method. Ahmed et al. [9] studied the three dimensional laminar boundary layer flow of a micropolar fluid due to a stretching surface, the analysis and discussion of stretching and material parameter have also been made. El-Kabeir et al. [10] analyzed the heat and mass transfer flow of a micropolar fluid, which is passing through a permeable continuously moving surface, two different cases: plane surface moving parallel to the free stream and surface moving opposite to the free stream are considered. Rahman and Al-Lawatia [11] founded the influence of higher order chemical reaction and heat transfer flow of micropolar fluid past a permeable stretching sheet in a porous medium, the Darcy parameter shows the decreasing rate of the surface mass. Turkyilmazoglu [12] studied the flow of a micropolar fluid due to a stretching sheet with heat transfer. Nadeem et al. [13] examined the unsteady two dimensional magnetohydrodynamics (MHD) boundary layer flow of incompressible micropolar fluid through a porous medium near a forward stagnation point of a plane wall. Ashraf and Batool [14] numerically studied the MHD and heat transfer flow of the axisymmetric micropolar fluid passed through the stretching disk.

In the history of fluid mechanics, the most significant article in the literature of fluid dynamics is Von Karman [15]. He was the first one who calculated the solution of the laminar flow of a viscous fluid over a rotating disk. This paper captivated the attention of many researchers due to its several technical and industrial applications. Many researchers reinvestigated this article in various dynamics with MHD effect, porous medium, chemical reactions, thermal effect, uniform suction/injection effects, etc. [16]. The fluid passing through porous media has many applications, such as aerogels, injection of mud's, porous rocks, alloys, slurries or cement grouts to reinforce soils, foams and foamed solids, micro emulsions, polymer blends, and most important application is drilling fluids through injection in rocks for the reinforcement of the wells and also for enhancing oil recovery, etc. [17,18]. In 1856, Darcy [19] proposed a Darcy law in which he established the relation between the pressure drop and flow rate relation in a porous medium. Tan and Masuoka [20,21] examined the Stokes' first problem for the second grade and Oldroyd-B fluids by using the modified Darcy's law. Hayat et al. [22] re-examined the Stoke's first problem for non-Newtonian forth order fluid by using the modified Darcy's law in a porous medium.

To the best of authors' knowledge, the micropolar fluid model has never been investigated for an off centered rotating disk in a porous medium with Darcy resistance effect. In this article, the problem of Wang [23] is extended to study the influence of micropolar fluid parameters (material parameter, spin gradient viscosity parameter, and micro-inertia density parameter) based on the vortex viscosity, spin gradient viscosity, the material constant, and micro-inertia of the micropolar fluid over an off centered rotating porous disk with modified Darcy law, which has not been investigated previously. To attain the analytical solution of the current problem, the similarity transformations from [23] are used to reduce the partial differential equations of motion into non-linear ordinary differential equations (ODEs). The ODEs are then solved by a well-known non perturbative