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Perturbation methods in evolutionary spectral analysis for linear dynamics and equivalent statistical linearization

Thomas Canor [a,b], Luca Caracoglia [c], Vincent Denoël [a,*]

[a] Structural Engineering Division, University of Liège, Allée de la Découverte, 9, B52/3, 4000 Liège, Belgium
[b] National Fund for Scientific Research of Belgium, Belgium
[c] Department of Civil and Environmental Engineering, Northeastern University, 400 Snell Engineering Center, 360 Huntington Avenue, Boston, MA 02115, United States

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ABSTRACT

The analysis of large-scale structures subject to transient random loads, coherent in space and time, is a classic problem encountered in earthquake and wind engineering. The simulation-based framework is usually seen as the most convenient approach for both linear and nonlinear dynamics. However, the generation of statistically consistent samples of an excitation field remains a heavy computational task. In light of this, perturbation techniques are applied to develop and improve evolutionary spectral analysis.

Advantageously performed in a standard modal basis, this evolutionary spectral analysis for linear structures requires the computation of the modal impulse response matrix. However, this matrix has no general closed-form expression in the presence of modal coupling. We propose therefore to model it by an asymptotic approximation, obtained by the inverse Fourier transform of an asymptotic expansion of the modal transfer matrix of the structure. This latter expansion considers the modal coupling as a perturbation of a main decoupled system. This strategy leads to an expansion known in a closed-form. Finally, the semi-group property allows the use of an efficient recurrence relation to approximate the modal evolutionary transfer matrix, i.e. the evolutionary extension of the transfer matrix.

The asymptotic expansion-based method and the recurrence relation are then applied to nonlinear transient dynamics by using Gaussian equivalent linearization. This extension is formalized by a multiple timescales approach, allowing to consider a linearized structure, namely a time variant system, as piecewise linear time invariant depending on a statistical timescale. The proposed developments are finally illustrated on realistic civil engineering applications.

1. Introduction

1.1. Context and methods

In civil engineering, structures are designed to resist random loadings, such as wind forces or ground acceleration during earthquakes. In some cases, these loadings are said to be nonstationary or transient, i.e. their statistical characteristics are time-dependent. This property is well-known for earthquakes, as the phenomenon is usually modeled by three phases: build-up, plateau and decay. Moreover, it would be fallacious to limit wind analysis to stationary processes. For example, in the case of *downbursts* or *thunderstorms*, we indubitably need to take into account the time evolution of the mean wind velocity and the intensity of turbulence. In the context of risk analysis in civil engineering, an adequate design requires to compute the time evolution of the statistics of the structural response.

The development of suitable methods to perform transient analysis has attracted the attention of the research community in the last 30 years. For linear structures, an analytical approach based on Duhamel's convolution [12], also called *pseudo-excitation approach* [38], may be used, even though this method proves to be exclusively efficient for systems in which the modal expansion is capable of decoupling the dynamic equations. Actually, practitioners and engineers consider Monte Carlo simulations as the most convenient method to perform linear nonstationary analysis. As to nonlinear structures, Proppe et al. [50] show that Monte Carlo simulations [19,59] and the equivalent linearization [52] are the only two *feasible* methods to perform stochastic analysis of large-scale structures, especially in a nonstationary setting.

Other time domain methods have been developed for both linear and nonlinear analyses. For instance, the concept of *stochastic integration scheme* proposed by To [64,65] has been

* Corresponding author.
E-mail addresses: thomas.canor@gmail.com (T. Canor), lucac@coe.neu.edu (L. Caracoglia), v.denoel@ulg.ac.be (V. Denoël).

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recently improved by Tootkaboni [66] and applied to linear systems subject to non-white excitations or to nonlinear systems by considering them as piecewise linear. The question of *nonstationary Gaussian equivalent linearization* (GEL) has been also addressed interestingly by the team of Schuëller in [46,54,55], who proposed a method based on the Karhunen–Loéve (K–L) expansion of the excitation. Accordingly, the K–L expansion of the modal state space vector of the structure is substituted into the linearized equation of motion. The K–L decomposition can also be used when measured data are available [60], even if they are often difficult to obtain in wind and earthquake engineering. Following similar ideas, stochastic averaging techniques have been used to determine approximate closed-form expressions for some specific problem, e.g. concerning the use of nonlinear viscous dampers [67].

A main drawback of the simulation-based framework in risk quantification is the difficulty to deal with large dimensional coherent excitation fields. In wind engineering, the forces due to wind blowing on large structures are usually modeled as a spatially coherent excitation field [61], i.e. a set of random processes simultaneously depending on both time and space [25]. On the other hand, for long structures subject to earthquake loads, the ground accelerations measured at different supports are different, but not statistically independent [35,72]. Actually, neglecting the coherence within the field may lead to underestimation of the structural response, while assuming fully correlated processes may result in possible overestimation. In fact, the computational burden inherent to these approaches is mainly associated with the generation of consistent and accurate samples of the excitation fields.

In earthquake or wind engineering, the coherence within the loading can be adequately modeled by a full power spectral density (PSD) matrix. The coherence is a way to express the time correlation in the frequency domain, a reason for trying to keep on working in the frequency domain. In wind engineering, a spectral approach is usually chosen to perform buffeting analysis, as explained for instance in [20,39]. For linear stationary problems, the spectral approach is by far the most efficient one, since it just consists in matrix multiplications performed for selected frequencies and in an integration of these matrices over the frequency domain. These operations are naturally preceded by a modal projection. In this context, the spectral approach clearly outscores the simulation-based framework in terms of accuracy and efficiency. In many cases, consideration of the different timescales of the excitation and of the response make this approach yet more efficient, even in case of slight structural nonlinearities [15].

This work aims at applying, in a nonstationary setting, the spectral analysis to both linear and nonlinear structures in civil engineering. Within this context, a specific family of transient processes is considered: the *evolutionary processes*. Such an unsteady process may be described by the PSD of an *embedded stationary process* and a modulation *time window*. Actually, the *evolutionary spectral* (EvSp) *analysis* may be understood as a natural extension of the spectral analysis, since the former approach should necessarily tend to the latter one over large timescales, provided the intensity envelope remains constant in time. The evolutionary approach remains an elegant formulation of transient phenomena, for which evolutionary models are available, as for earthquakes or downbursts [68,33,10,11,56,73]. However, it requires to work in both time and frequency domains, contrary to the classical spectral analysis.

As source and ground, the *stricto sensu* EvSp analysis has been formalized by Priestley [48,49]. The first pertinent applications are due to Hammond for SDOF and MDOF systems [28,29]. Those contributions are only focused on linear time-invariant (LTI) systems. Although the spectral approach is recognized as the most suitable method to analyze large-dimensional linear structures subject to stationary loadings, the evolutionary spectral analysis has not encountered a real enthusiasm in the fields of engineering for some reasons explained hereinafter. Before, some mathematical statements about evolutionary processes are presented.

1.2. Mathematical statements

On a probability space $(\Theta, \mathfrak{F}, \mathbb{P})$, the equation of motion of a n-DOF nonlinear system is

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} + \mathbf{g}(\mathbf{y}, \dot{\mathbf{y}}) = \mathbf{f}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the n-dimensional mass, damping and stiffness matrices of the system, respectively, $\mathbf{f}(t, \theta)$: $\mathbb{R}^+ \times \Theta \mapsto \mathbb{R}^n$ is the vector of random exogenous Gaussian forces and the dot symbol denotes the time derivative. The hypothesis of Gaussianity is used throughout this work, since random loading processes can often be described in this way in wind or earthquake engineering. The vector $\mathbf{y}(t, \theta)$: $\mathbb{R}^+ \times \Theta \mapsto \mathbb{R}^n$ gathers the nodal displacements expected to be non-Gaussian processes due to the nonlinear forces in the vector function $\mathbf{g}(\mathbf{y}, \dot{\mathbf{y}})$: $\mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}^n$. With this formalism, the equation of motion is split into four contributions: inertial forces, internal linear forces, internal nonlinear forces and exogenous random forces. Discarding the nonlinear forces $\mathbf{g}(\mathbf{y}, \dot{\mathbf{y}})$ in (1) leads to a linear governing equation, referred to as the *linear subsystem* in the sequel.

In Eq. (1), the components of $\mathbf{f}(t)$ are supposed to be *evolutionary processes*. These random processes belong to the family of nonstationary processes, widely used in civil and mechanical engineering. Formally, the spectral distribution of the nodal forces is such that the vector of forces may be written in the form of a Fourier–Stieltjes integral, like

$$\mathbf{f}(t) = \int_{\mathbb{R}} e^{\imath\omega t}\mathbf{a}(t, \omega)\mathrm{d}\tilde{\mathbf{f}}(\omega) \tag{2}$$

with $\imath = \sqrt{-1}$ and $\mathbf{a}(t, \omega)$: $\mathbb{R}^+ \times \mathbb{R} \mapsto \mathbb{R}^{n\times n}$ being a diagonal matrix gathering deterministic *time windows* (also called *intensity functions* or *time envelopes*) and $\tilde{\mathbf{f}}(\omega, \theta)$: $\mathbb{R} \times \Theta \mapsto \mathbb{C}^n$ being the vector of spectral processes related to the embedded stationary processes. This formulation is used by Priestley on the basis of the developments of Bartlett [2].

In a more particular case, but very often encountered in practice, all the stationary forces on a structure are modulated by the same time window $a(t, \omega)$. Most of the time, this assumption is used in seismic engineering. In wind engineering, the use of a matrix $\mathbf{a}(t, \omega)$ allows to model wind direction evolution during nonstationary storms.

Assuming $\mathbf{f}(t)$ a zero-mean process, the time-dependent covariance function of $\mathbf{f}(t)$, noted $\boldsymbol{\Sigma}_{\mathbf{f}}(t)$, is given by

$$\boldsymbol{\Sigma}_{\mathbf{f}}(t) = \int_{\mathbb{R}} \hat{\mathbf{S}}_{\mathbf{f}}(t, \omega)\mathrm{d}\omega \tag{3}$$

with $\hat{\mathbf{S}}_{\mathbf{f}}(t, \omega)$: $\mathbb{R}^+ \times \mathbb{R} \mapsto \mathbb{C}^{n\times n}$ being the evolutionary PSD of $\mathbf{f}(t)$. The hat used to denote evolutionary PSD stems from [39] to avoid any possible confusion with the PSD of the embedded stationary processes $\mathbf{S}_{\mathbf{f}}(\omega)$. Assuming that $\tilde{\mathbf{f}}(\omega)$ is a random process with *orthogonal increments* [25,39], $\hat{\mathbf{S}}_{\mathbf{f}}(t, \omega)$ is

$$\hat{\mathbf{S}}_{\mathbf{f}}(t, \omega) = \mathbf{a}(t, \omega)\mathbf{S}_{\mathbf{f}}(\omega)\mathbf{a}^T(t, \omega). \tag{4}$$

This previous equation summarizes the general philosophy of the evolutionary spectral representation: an embedded stationary process mainly described by its cross-PSD matrix and a modulation matrix introducing the time dependency in the problem. For the sake of simplicity and clarity in the following developments,