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ORIGINAL ARTICLE

Solution of analytical model for fuel spray penetration via homotopy perturbation method

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Abstract The present paper attempts to solve equations in the initial stage and the two-phase flow regime of fuel spray penetration using the HPM-Padé technique, which is a combination of the homotopy perturbation method (HPM) and Padé approximation. At the initial stage, the effects of the droplet drag and the air entrainment were explained while in the two-phase flow stage, the spray droplets had the same velocities as the entrained air. The results for various injection pressures and ambient densities are presented graphically and then discussed upon. The obtained results for these two stages show a good agreement with previously obtained expressions via successive approximations in the available literature. The numerical result indicates that the proposed method is straight forward to implement, efficient and accurate for solving nonlinear equations of fuel spray.

1. Introduction

Non-linear phenomena play a crucial role in applied mathematics and physics. It is known that most of the engineering problems are non-linear, thus it is difficult to solve them analytically.

Many new analytical techniques have been successfully developed by diverse groups of mathematicians and physicists, such as, perturbation method [1], homotopy perturbation method [2], modified homotopy perturbation method [3], Rational homotopy perturbation method [4], He's homotopy perturbation method [5], modified He's homotopy perturbation method [6], optimal iteration perturbation method [7], generalization of modified differential transforms method [8], and other new methods [9–11].

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One of the recent analytical methods used in the literature, namely, the homotopy perturbation method (HPM) which was firstly proposed by Chinese mathematician He [12] has attracted special attention of researchers as it is flexible in applying and gives sufficiently accurate results with modest effort. This method is a powerful series-based analytical tool that has been used by many authors. however, the convergence region of the obtained truncated series approximation is limited and in the best case scenario, it needs some enhancements to enlarge the convergence region of the approximate solution. It is well known that Padé approximations which was presented by Padé [13], have the advantage of manipulating the polynomial approximation into a rational function of polynomials. This manipulation provides us more information about the mathematical behavior of the solution. So, the applying of Padé approximations to the truncated series solution obtained by HPM will be an effective way to enlarge the convergence domain and greatly improve the convergence rate of the truncated power series. In recent years, the HPM-Padé method has been successfully employed to solve many types of nonlinear problems [14,15].

Spray penetration is one of the most important parameters that influences vapor distribution, vapor mixing with air, and gas turbulence in the combustion chamber. In particular, fuel spray penetration (FSP) has practical applications ranging from agricultural sprays to sprays in machineries such as boilers, diesel engines, gas turbines, and space rockets. Further, FSP has great impact on the efficiency and thrust power of engines. The prediction of the flow proper ties of fuel spray in a combustion engine requires consideration of two phases in the flow field. This is because the turbulence inside the cylinder controls the mixing of the fuel with air. The combustion of FSP has been extensively studied experimentally and theoretically [16–19]. A detailed study on fundamentals of the engine sprays would gain more insight when the various phenomena such as the formation of ligaments and their break-up, droplet break-up and evaporation, the entrainment of air and the effects of turbulence [20,21] (just to mention a few) are to be taken into consideration. On many occasions, therefore, it is far more important to establish a hierarchy of the importance of the various processes and develop simplified models suitable for practical applications. In this light, Sazhin et al. [22] have developed simple analytical models describing the initial stage of spray penetration in three flow regimes viz: Stokes, Allen and Newton, respectively.

In this paper, the authors are interested in applying the HPM as a powerful series-based analytical tool by enhancing it with the Padé approximants, which eventually improve the results. This is accomplished by increasing the accuracy and enlargement of intervals of convergence for the solution of the equations that govern the initial stage and the two-phase flow regime of the fuel spray penetration. The solutions are compared with those derived by Sazhin [22] and Ebrahimian [23] and the well-known fourth order Runge-Kutta method in order to verify the exactness of the HPM-Padé technique results.

2. The initial stage

2.1. Mathematical formulation

The velocities of droplets injected from a nozzle are initially much greater than velocity of the gas (air) stream, but are slowed down due to the drag force, while gas is accelerated. The most general equation describing the dynamics of an individual droplet can be written as

$$m_d \frac{dv_d}{dt} = -\frac{1}{2} C_D \rho_g \left(v_d - v_g\right)^2 A_d, \tag{1}$$

where m_d, v_d and A_d are droplet's mass, velocity and cross-sectional area, respectively, v_g and ρ_g are gas velocity and density, respectively. In this study, only one-dimensional dynamics of gas and droplets are considered. "C_D" is the drag coefficient, which depends on the shape of the droplet and the Reynolds number: $Re = 2\rho_g(v_d\text{-}v_g)r_d/\mu_g$. r_d and μ_g denote droplet's radius and gas dynamic viscosity, respectively.

Under assumption that the droplets are perfect spheres, then Eq. (1) is simplified to

$$\frac{d^2 s}{dt^2} = -\frac{3}{8} r_d C_D \frac{\rho_g}{\rho_d} \left(\frac{ds}{dt} - v_g\right)^2, \tag{2}$$

where ρ_d is the droplet's density, s the distance measured from the nozzle, and $v_d = ds/dt$. Eq. (2) is not easily amenable to analytical results, especially due to the drag coefficient C_D, which is a rather complicated function of the Reynolds number. In this regard, a number of approximations of C_D have been suggested in the literature [22]. The most convenient approximation is the one found in Douglas et al., which considered 3 ranges of Reynolds numbers: $Re \leq 0.2$ (Stokes flow), $0.2 < Re \leq 500$ (Allen flow), and $500 < Re \leq 105$ (Newton flow). The functions C_D (Re) for these flows are given by the expressions: $C_D = 24/Re$ (Stokes flow), $C_D = 18.5/Re^{0.6}$ (Allen flow), and $C_D = 0.44$ (Newton flow). The expressions for C_D do not take into account effects of droplet acceleration, internal circulation, vaporization, burning, non-spherical shape, vibrations, and heating processes.

Two basic approximations were made to Eq. (2) in relation to v_g, namely: (i) $v_g \ll v_d$, and (ii) $v_g = k\sqrt{s}$, where $k = \sqrt{K/r_{g0}}$, and $K = 3r_s^2 \alpha_d/(8r_d) C_D \nu_{d0}^2 = \text{constant}$. Here, v_{d0} is the initial droplet velocity.

The approximate analytical results emanating from these approximations and their physical interpretations are extensively discussed in Sazhin et al. [22].

Now with the latter approximation for v_g $(= k\sqrt{s})$ in the Eq. (15), the governing equations for Stokes, Allen and Newton flows by Refs. [22,23] are respectively written as follows:

$$\frac{d^2 s}{dt^2} + \alpha \frac{ds}{dt} - \alpha k \sqrt{s} = 0, \tag{3a}$$

$$\frac{d^2 s}{dt^2} + \beta \left(\frac{ds}{dt}\right)^{1.4} - 1.4 \beta k \sqrt{s} \left(\frac{ds}{dt}\right)^{0.4} = 0, \tag{3b}$$