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ORIGINAL ARTICLE

Unsteady thin film flow of a fourth grade fluid over a vertical moving and oscillating belt

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Abstract This article studies the unsteady thin film flow of a fourth grade fluid over a moving and oscillating vertical belt. The problem is modeled in terms of non-nonlinear partial differential equations with some physical conditions. Both problems of lift and drainage are studied. Two different techniques namely the adomian decomposition method (ADM) and the optimal homotopy asymptotic method (OHAM) are used for finding the analytical solutions. These solutions are compared and found in excellent agreement. For the physical analysis of the problem, graphical results are provided and discussed for various embedded flow parameters.

1. Introduction

Due to the diverse physical structures of non-Newtonian fluids, several constitutive models have been proposed in the literature to predict all of their salient features. Generally, there are three non-Newtonian fluids models. They are known as (i) the differential type, (ii) the rate type, and (iii) the integral type. However, the most famous amongst them are the first two models. In this article, we will study the first model, the differential type fluid also known as grade n fluids and consider its subclass known as the fourth grade fluid. The simplest subclass of differential type is the second grade fluid for which one can reasonably hope to obtain an analytical solution. Although a second grade fluid model also known as grade

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2 is able to predict the normal stress differences but it does not take into account the shear thinning and shear thickening phenomena that many fluids exhibit [1–4]. On the other hand, a third grade fluid model (grade 3 fluid) represents a more realistic description of the behavior of non-Newtonian fluids. This model is known to capture the non-Newtonian effects such as shear thinning or shear thickening as well as normal stresses [5–8]. There is another model for differential type fluids known as fourth grade fluid model (grade 4 fluid) which at one time capture most of the non-Newtonian flow properties. Means, Cauchy stress tensor appearing in this fluid model contains several parameters. Due to which the governing equation for fourth grade fluid becomes quite complicated. This model is also very capable to predict the effects of normal stresses that lead to phenomena like rod-climbing and die-swell [9]. Therefore, very few studies, which illustrates the effect of fourth grade fluids even on steady flow, have been reported in the literature. In general, it is not easy to study fourth grade fluid even for steady flow problems. However, under certain assumptions based on the flow situation, it is possible and some investigations are carried out in this direction [10–14]. On the other hand, the science of thin liquid films has developed rapidly in recent years. Thin film flow problems appear in many fields, varying from specific situations in the flow in human lunges to lubrication problems in engineering which is probably one of the largest subfield of thin film flow problems [15]. Few other applications are found in coating flows, biofluids, microfluidic engineering, and medicine [16,17]. Two well know examples from everyday life are slipping on a wet bathroom floor and aquaplaning or hydroplaning on a wet road. The study of thin film flow for practical applications is a challenging interplay between fluid mechanics, structural mechanics and theology. Based on this motivation, recently several researchers are getting interested to study the thin film flows. Amongst them, we discuss here some important contributions of the following researchers.

Siddiqui et al. [18,19] discussed the analytical solution of thin film flow of third grade fluid and Oldroyed-8 constant fluid through vertical moving belt by applying adomian decomposition method (ADM) and variational iteration method (VIM). Aiyesimi et al. [20] investigated the unsteady magnetohydrodynamic (MHD) thin film flow of a third grade fluid with heat transfer and no slip boundary condition down an inclined plane. Gul et al. [21,22] studied analytically the thin film flows of second grade and third grade fluids through a vertical belt for lift and drainage problems using optimal homotopy asymptotic method (OHAM) and ADM. Few other investigations in this direction are mentioned in Refs. [23–40].

The basic theme of this article is to venture further in the regime of thin film flows and to extend this idea to the fourth grade fluid executing the unsteady motion over a vertical oscillating belt using two analytical techniques namely the OHAM and ADM methods. OHAM on the other hand, is a new powerful technique which is a generalized form of homotopy perturbation method (HPM) and HAM. It is a straightforward technique and does not require the existence of any small or large parameter as does the traditional perturbation method. OHAM has successfully applied to a number of nonlinear problems arising in the science and engineering by various researchers. This proves the validity and acceptability of OHAM as a useful technique. For the correctness and verification of OHAM solutions, they are compared with those obtained by ADM. The rest of the paper is arranged as follows: The constitutive equations for fourth grade fluid are given in Section 2. Using these constitutive equations for unidirectional and one dimension flow, the continuity equation is satisfied identically and the momentum equation is derived in Section 3 with some physical conditions. In Section 4, the basic concepts of ADM and OHAM are discussed. These methods are used in Section 5 where the analytical solutions for both lifting and drainage problems are obtained. The graphical results are given in Section 6. This paper ends with some conclusions in Section 7.

2. Fundamental equation

The constitutive equations for fourth grade fluid model is

$$\boldsymbol{T} = -p\boldsymbol{I} + \mu\boldsymbol{A}_1 + \sum_{i=1}^{3}\boldsymbol{S}_i, \tag{1}$$

where $\boldsymbol{T}$ is the Cauchy stress tensor, $\boldsymbol{I}$ is identity tensor, p is fluid pressure and $\boldsymbol{S}_i$ is the extra stress tensor

$$\begin{aligned}
\boldsymbol{S}_1 &= \alpha_1\boldsymbol{A}_2 + \alpha_2\boldsymbol{A}_1^2,\\
\boldsymbol{S}_2 &= \beta_1\boldsymbol{A}_3 + \beta_1(\boldsymbol{A}_1\boldsymbol{A}_2 + \boldsymbol{A}_2\boldsymbol{A}_1) + \beta_3\left(\mathrm{tr}\boldsymbol{A}_2^2\right)\boldsymbol{A}_1,\\
\boldsymbol{S}_3 &= \gamma_1\boldsymbol{A}_4 + \gamma_2(\boldsymbol{A}_3\boldsymbol{A}_1 + \boldsymbol{A}_1\boldsymbol{A}_3) + \gamma_3\boldsymbol{A}_2^2\\
&\quad + \gamma_4(\boldsymbol{A}_2\boldsymbol{A}_1^2 + \boldsymbol{A}_1^2\boldsymbol{A}_2) + \gamma_5(\mathrm{tr}\boldsymbol{A}_2)\boldsymbol{A}_2 + \gamma_6(\mathrm{tr}\boldsymbol{A}_2)\boldsymbol{A}_1^2\\
&\quad + \{\gamma_7(\mathrm{tr}\boldsymbol{A}_3) + \gamma_8(\mathrm{tr}\boldsymbol{A}_2\boldsymbol{A}_1)\}\boldsymbol{A}_1.
\end{aligned} \tag{2}$$

Here μ is coefficient of viscosity,

$\alpha_i(i=1,2)$, $\beta_j(j=1,2,3)$ and $\gamma_k(k=1,2,\ldots,8)$

are material constants of second, third and fourth grades respectively. Rivlin–Ericksen tensors $\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3$ and $\boldsymbol{A}_4$ are defined as:

$$\boldsymbol{A}_0 = \boldsymbol{I},\ \boldsymbol{A}_1 = (\nabla\boldsymbol{V}) + (\nabla\boldsymbol{V})^{\mathrm{T}}, \tag{3}$$

$$\boldsymbol{A}_n = \frac{D\boldsymbol{A}_{n-1}}{Dt} + \boldsymbol{A}_{n-1}(\nabla\boldsymbol{V}) + (\nabla\boldsymbol{V})^{\mathrm{T}}\boldsymbol{A}_{n-1}, \quad n = 2,3,4. \tag{4}$$

The continuity and momentum equation for incompressible fluid are given by