Applied Ocean Research 79 (2018) 88–100

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

A unified coupled-mode method for wave scattering by rectangular-shaped objects

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ARTICLE INFO

Keywords:
Permeable breakwater
Normal incident wave
Reflection/Transmission coefficient
Eigenfunction expansion

ABSTRACT

This paper presents a general solution for wave scattering by stationary objects, which consist of a submerged rectangular plate and a floating rectangular dock. These objects have the same width and their centerlines are aligned. The objects can be either permeable or solid. Within the framework of linear potential flow theory, the method of eigenfunction expansion is adopted to solve the wave scattering problem. Two auxiliary potentials are introduced in each flow regions to facilitate the eigenfunction expansions. This approach avoids solving a complex dispersion relation. A computing program has been developed based on the present general solutions. The numerical model is checked with existing solutions for single rectangular object, such as a surface-piercing permeable breakwater, a bottom mounted submerged permeable or solid breakwater and a submerged permeable plate. Perfect agreements between the present solutions for the general model and the existing solutions are observed. The general model can also yield new solutions for different combinations of objects, e.g.,a single floating permeable breakwater, and a combination of a floating and a bottom mounted permeable or solid breakwater.

1. Introduction

Permeable structures such as surface-piercing and submerged rubble mound breakwaters are often used as a means for protecting shorelines and providing sheltered area for sea vessels. For decades, analytical approaches for assessing the effectiveness of these permeable structures have been developed by many researchers. Sollitt and Cross [1] introduced a classical mathematical model for describing the effects of permeable object on wave scattering. Permeable object can be characterized by three dimensionless parameters, representing the linearized resistance coefficient, the inertial coefficient and the porosity. Based on this mathematical formulation, numerous studies have been performed (e.g. [2–5],). It is clear that a permeable object can be converted into a solid object by specifying the porosity to be zero. On the other hand, the permeable object can be removed by specifying the porosity to be unity.

One of the popular approaches for estimating wave reflection and transmission by a submerged permeable breakwater employs the method of eigenfunction expansion and assumes that the complex wave numbers (eigenvalues) in the regions above and inside the permeable breakwater are the same (e.g., [3,4]). Although this assumption enables the analysis, it still results in a complex dispersion relation; solving this complex transcendental equation is non-trivial. Moreover, although the simplifying assumption is necessary for obtaining the solution, it does not appear to have a sound physical justification. Avoiding this seemly unclear assumption, Lee and Liu [6] presented a new analytical solution for the submerged permeable breakwater problem using a different eigenfunction expansion method. Combining Lee and Liu [6]'s approach and the approach originally developed by Mei and Black's [11] for impermeable breakwater, Liu et al. [7] and Liu and Li [8] presented analytical solutions for wave scattering by a submerged horizontal permeable plate with finite thickness and by a surface-piercing permeable breakwater, respectively. In Mei and Black's [11] approach only half of physical domain is solved, which simplifies the analysis.

The primary objective of this paper is to derive a general analytical solution for wave scattering by multiple rectangular shaped objects, which consist of a rectangular plate and a rectangular floating dock. These two objects have the same width and their centerlines are aligned. The general solution aims to cover all the existing wave scattering solutions for a single object, such as a surface-piercing breakwater, bottom-mounted breakwater, and submerged plate. In addition, the general solution can be reduced to other new configurations, such

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https://doi.org/10.1016/j.apor.2018.07.008
Received 16 April 2018; Received in revised form 15 June 2018; Accepted 13 July 2018

as a single floating permeable breakwater and a combination of a floating and a bottom mounted permeable or solid breakwater. It is reiterated here that the objects can be either impermeable or solid. A computing program has been developed based on the general solution with the features of simple implementation, accuracy and robustness. The secondary objective of the paper is to examine if there are any differences between the solutions for a submerged permeable breakwater obtained from the present approach and the existing solutions by Rojanakamthorn et al. [3].

The paper is structured as follows. The mathematical formulation of the wave scattering problem will be described in the following section. In Section 3, using the eigenfunction expansion method and introducing two auxiliary potentials, an analytical solution is presented. The resulting matrix equations are numerically solved. In Section 4, existing analytical results for different types of single breakwater are shown to be special cases of the present general solutions. Solutions for two new types of breakwater configuration are produced by the present model. In Section 5, the existing solutions for a submerged permeable breakwater proposed by Rojanakamthorn et al. [3] are examined and are compared with the present solutions. The fundamental differences between the existing solutions and the new solutions are discussed. Finally, concluding remarks are provided in Section 6.

2. The boundary value problem

The geometry of the wave scattering problem is illustrated in Fig. 1. A combination of a floating permeable breakwater and a submerged permeable plate is considered. Both of them are stationary. The floating breakwater has a draft "d", while the thickness of the permeable plate is "a". The edges of these two objects are aligned with the same width "$2b$". The space between the floating breakwater and the submerged plate is denoted by "c", while the space below the plate and the seabed is denoted by "e." The water depth, "h", is assumed to be a constant. A Cartesian coordinate system is employed, where x-axis is in the direction of wave propagation and z-axis points upwards with the origin being at the intersection of the still water level and the centerline of the breakwaters. It is obvious that one can manipulate the dimensions of these two objects to create many different configurations. For example, a single surface-piercing breakwater can be reproduced by shrinking both "c" and "e" to zero, and removing the permeable plate.

Following Sollitt and Cross [1], the physical characteristics of the permeable objects can be described by the porosity ε, the inertial coefficient s, and the linearized friction coefficient f. The inertial coefficient s is defined as $s = 1 + \frac{1-\varepsilon}{\varepsilon}C_m$, where C_m is the added mass coefficient. Therefore, s is greater than or equal to one. The linearized friction coefficient f depends on the porosity, the intrinsic permeability as well as the fluid viscosity. In general, the friction coefficient is flow-dependent and should be treated as a time varying parameter. However, for simplicity, it is assumed to be a constant in this paper. For practical problems, it usually takes on values between 0 and 10 ([9,10,5]). We remark here that the permeable object can be converted to a solid when the porosity becomes zero ($\varepsilon = 0$). On the other hand, the permeable object can be removed by assigning porosity be unity, $\varepsilon = 1$. Thus, the present solutions are also applicable to solid breakwaters and to combinations of permeable and impermeable breakwaters with different configurations. Some of them will be demonstrated in a later section.

Consider a small amplitude, simple harmonic incident wave train, propagating in the positive x-direction. Following Sollitt and Cross' [1] formulation, the fluid motions inside and outside the permeable objects can be described by a potential function $\Phi_j(x,z,t)$, where the subscript j denotes the jth region (j = 1–6) as shown in Fig. 1. Denoting the angular frequency as ω, the potential function in each region can be further expressed as $\Phi_j(x,z,t) = \mathrm{Re}[\phi_j(x,z)\, e^{-i\omega t}]$, where Re[.] represents the real part of the argument; $i = \sqrt{-1}$; t is the time and $\phi_j(x, z)$ represents the spatial distribution of the potential function.

To satisfy the continuity the spatial potential function in each region must satisfy the Laplace equation:

$$\frac{\partial^2\phi_j}{\partial x^2} + \frac{\partial^2\phi_j}{\partial z^2} = 0,\ \ (j = 1\text{–}6) \tag{1}$$

The boundary conditions on the still water level, $z = 0$, and the seabed, $z = -h$, respectively, can be expressed as:

$$\frac{\partial\phi_j}{\partial z} = K\phi_j,\ \ (j = 1 \text{ and } 6) \text{ on } z = 0, \tag{2}$$

$$\frac{\partial\phi_2}{\partial z} = K(s_1 + if_1)\phi_2,\ \ \text{on } z = 0, \tag{3}$$

$$\frac{\partial\phi_j}{\partial z} = 0,\ \ (j = 1,\ 5,\ 6) \text{ on } z = -h, \tag{4}$$

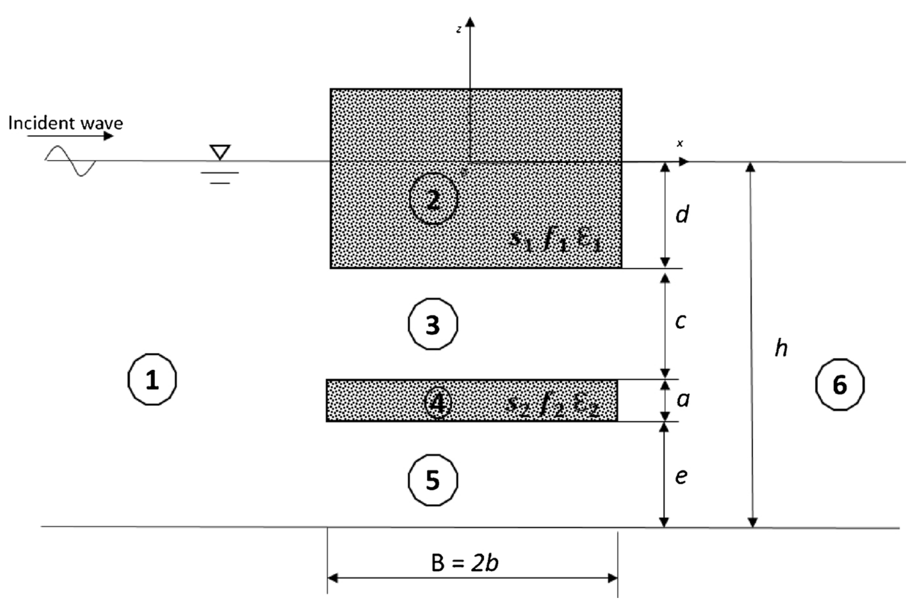

Fig. 1. Sketch of wave scattering by a combination of a floating permeable breakwater and a submerged horizontal permeable plate with finite thickness. Both objects are stationary.