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Stochastic nonlinear ship rolling in random beam seas by the path integration method

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ABSTRACT

Nonlinear ship rolling in random seas is a serious threat to ship stability. In this work, the dynamic stability of the vessel in random seas is evaluated by means of probabilistic methods. Specifically, the Markov diffusion theory is applied in order to study the stochastic aspects of the roll motion driven by random wave loads. The roll motion is modeled as a single-degree-of-freedom (SDOF) model in which the nonlinear damping and restoring terms as well as the random wave excitation are all incorporated. The stationary wave excitation moment in the SDOF model is represented as a filtered white noise by employing a second order linear filter. Therefore, a four-dimensional (4D) Markov dynamic system is established by combing the SDOF model with the linear filter model. Because the probabilistic property of the 4D Markov system is governed by the Fokker–Planck (FP) equation, the response statistics of roll motion can be obtained by solving the FP equation via an efficient 4D path integration (PI) method, which is based on the Markov property of the coupled dynamic system. Furthermore, the random wave excitation is approximated by an equivalent Gaussian white noise, and a two-dimensional (2D) PI technique is applied in order to obtain the response statistics of the dynamic system driven by this Gaussian white noise. The rationality and accuracy of applying the equivalent Gaussian white noise to simulate nonlinear ship rolling in random seas is studied. Moreover, the accuracy of the response statistics computed by the 4D and 2D PI techniques is verified by means of the versatile Monte Carlo simulation technique.

1. Introduction

Ship sailing in random beam seas is considered as the worst condition with respect to navigation ability. Under such circumstance, roll resonance caused by wave excitation may lead to large amplitude roll motion or even capsizing. Nonlinear roll motion in random seas is one of the main reasons leading to stability failure. However, the current criteria of the International Maritime Organization (IMO) for evaluation of the intact stability are simply based on hydrostatic analysis [1]. There is no consideration with respect to the dynamic behaviors associated with the nonlinearities of roll damping and restoring terms as well as the randomness of wave excitation, which are important for stability assessment. With the awareness of the deficiencies of the current criteria for dynamic stability evaluation, the IMO is currently developing the next generation of intact stability criteria with a certain consideration of the physics associated with the dynamics of nonlinear roll motion and the randomness of wave excitation and roll response. For the direct assessment of ship dynamic stability in the framework of the next generation of intact stability criteria currently being developed, elaborate theoretical models as well as appropriate mathematical techniques are essential.

In this work, the dynamic stability in random seas is evaluated by means of a probabilistic approach, which means that the nonlinear stochastic roll response is in focus. Actually, prediction of the stochastic roll response is crucial for reliability based design and operation in practice, and also the response statistics are commonly used to characterize the dynamics of a random system. For the case of beam seas, the rolling ship is described as a single-degree-of-freedom (SDOF) system subjected to random external forcing and the nonlinearities associated with the damping and restoring terms are incorporated. However, for such nonlinear models, assessing the statistics of high-level response and the corresponding low probability levels is a difficult task, and limited progress has been made in the past decades [2].

The methodology based on time domain Monte Carlo simulation is the most commonly used technique for investigation of random systems as well as for evaluation of the stochastic

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responses. For example, the versatile Monte Carlo simulation technique can be applied to gain information about the spectral components and probability density functions of the response [3], to estimate the upcrossing rate and exceedance probabilities, etc. Moreover, for nonlinear ship rolling in random seas, Monte Carlo simulation is attractive in the sense that the nonlinear damping and restoring terms as well as the time-dependent random wave excitation can be directly dealt with. However, Monte Carlo simulation is only a brute force method and its accuracy and associated computational efficiency for prediction of the extreme responses would be a main drawback.

Besides the straightforward Monte Carlo simulation, Markov diffusion theory is attractive and popular for stochastic dynamic analysis of nonlinear systems since the probabilistic property of the dynamic system is governed by the Fokker–Planck equation. Because the Markov model is only valid for a system driven by Gaussian white noise or filtered white noise processes, a second-order linear filter is introduced in order to approximate the stationary wave excitation process [4]. Therefore, a four-dimensional (4D) augmented dynamic system is created by coupling the original SDOF model, a second-order nonlinear differential equation, with the linear filter employed to model the random wave excitation process as a filtered white noise process. The justification of applying the Markov diffusion theory for nonlinear dynamic systems subjected to random wave excitation has been mentioned in Naess and Johnsen [5]. For the extended Markov system, a host of useful response statistics can be obtained by solving the corresponding FP equation.

However, for the nonlinear coupled dynamic system associated with the high-dimensional FP equation, direct numerical solutions of the FP equation, e.g. by means of the finite-element method [6] and the finite difference method [7], are hardly feasible because of the so-called “curse of dimension” problem, i.e. difficulties arise due to the processing and storage needed for the computation. In addition the accuracy is hard to assess. As an alternative, the path integration (PI) method is applied in order to calculate the stochastic response of such extended dynamic systems. For the PI method, which is based on the Markov property of the dynamic system, no attempt is made to solve the FP equation directly and the evolution of the response probability density function (PDF) is calculated via a step-by-step solution technique invoking the total probability law. Specifically, the response PDF at a given time instant can be obtained when the response PDF at an earlier close time instant as well as the conditional PDF are already known. The PI method can provide high accuracy solutions to a range of problems, e.g. three dimensional (3D) PI procedures have been applied in Naess and Johnsen [5] and Karlsen [8] to estimate the response statistics of moored structures subjected to random wave loads. Iourtchenko et al. [9] studied the response PDFs of strongly non-linear SDOF systems excited by additive Gaussian excitation via a two dimensional (2D) PI technique, etc.

So far, only low-dimensional PI procedures have been applied to the area of nonlinear ship rolling. Lin and Yim [10] and Falzarano et al. [11] treated the wave excitation as regular waves perturbed by Gaussian white noise and evaluated the probability distributions of roll response by 2D PI algorithms. Chai et al. [12] studied the roll response and the associated reliability property of a vessel excited by Gaussian white noise via a 2D PI procedure and first-passage theory. Kougioumtzoglou and Spanos [13] recently developed a one-dimensional (1D) path integral framework, which is based on the statistical linearization and stochastic averaging method, for determining the stochastic response and the first-passage PDFs of ships rolling under random loads. However, these researches include certain simplifications in modeling the random wave excitation. In this paper, the feasibility of simplifying the random wave excitation as an equivalent Gaussian white noise process for prediction of the stochastic roll response will be studied.

In the present paper, the random wave excitation will be modeled as a filtered white noise via a second-order linear filter and an equivalent Gaussian white noise. Then, the 4D PI and 2D PI methods will be applied in order to evaluate the stochastic roll responses of the corresponding dynamic system. The comparison of the statistics of roll response excited by the filtered white noise with that excited by the corresponding equivalent Gaussian white noise will demonstrate serious deficiencies in the application of the latter excitation to simulate nonlinear ship rolling in random seas.

2. Modeling aspects

2.1. Equation of rolling motion

For the uncoupled roll motion in random beam seas, the governing equation is given by the following SDOF model:

$$(I_{44} + A_{44})\ddot{\theta}(t) + B(\dot{\theta}(t)) + \Delta \mathrm{GZ}(\theta(t)) = M(t) \tag{1}$$

where $\theta(t)$ and $\dot{\theta}(t)$ are the roll angle and the roll velocity, respectively. I_{44} is the moment of inertia, A_{44} represents the added mass coefficient. $B(\dot{\theta}(t))$ is the damping moment term and $\Delta\mathrm{GZ}(\theta(t))$ is the restoring moment term. $M(t)$ denotes the random wave excitation moment.

The damping moment term, consisting of wave radiation, viscous and vortex shedding components, is difficult to quantitatify, because these components are coupled with each other. Nevertheless, there are two empirical damping models commonly used to describe the roll damping term [14].

The linear-plus-quadratic damping (LPQD) model has been verified by numerous studies of experimental data [15] and it is expressed as:

$$B(\dot{\theta}(t)) = B_{44l}\dot{\theta}(t) + B_{44q}\dot{\theta}(t)|\dot{\theta}(t)| \tag{2}$$

where B_{44l} and are B_{44q} linear and quadratic damping coefficients, respectively.

On the other hand, the LPQD model is only once continuously differentiable and mathematically inferior to the linear-plus-cubic damping (LPCD) model [16], which is infinitely differentiable and given by the following cubic polynomial:

$$B(\dot{\theta}(t)) = B_{44l}\dot{\theta}(t) + B_{44c}\dot{\theta}^3(t) \tag{3}$$

in which, B_{44c} is the cubic damping coefficient.

The restoring term is expressed in terms of the displacement Δ and the restoring arm GZ, which can be obtained from standard hydrostatic software. The restoring arm is usually given by a nonlinear odd function of the roll angle, i.e.

$$\mathrm{GZ}(\theta) = C_1\theta - C_3\theta^3 \tag{4}$$

where C_1 and C_3 are the linear and nonlinear roll restoring coefficients of the restoring arm. It should be noted that the roll motion has a softening characteristic since the nonlinear restoring term is negative. For the softening cases, ship capsizing would occur when the roll angle exceeds the angle of vanishing stability beyond which the restoring moment becomes negative.

As for the wave excitation moment, it is a stationary Gaussian process which is described by the wave excitation moment spectrum, $S_{MM}(\omega)$. The latter is related to the wave energy spectrum, $S_{\xi\zeta}(\omega)$, by the following relationship [17]:

$$S_{MM}(\omega) = |F_{roll}(\omega)|^2 S_{\xi\xi}(\omega) \tag{5}$$