Applied Ocean Research 77 (2018) 69–77

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

Investigation on underwater positioning stochastic model based on acoustic ray incidence angle

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ARTICLE INFO

Keywords:
Stochastic model
Incidence angle
Underwater positioning
Accuracy

ABSTRACT

High-precision underwater positioning requires more accurate stochastic model. The commonly used equal weight stochastic model (EWSM) cannot be effective in precise underwater positioning process under the assumption of equal-precision observation. In this paper, inaccurate stochastic model effect on the positioning precision is analyzed and a novel acoustic ray incidence angle stochastic model (IASM) is proposed by taking the ranging error and the uncorrected acoustic ray bending error into consideration to improve positioning accuracy. The performance of both EWSM and IASM involving four different forms (the general proportional form, the cosine form, the exponent form and the piecewise cosine form) is tested by the simulation and experiment. The simulation results show that positioning accuracy for underwater static objects is 0.386 m when adopting EWSM while positioning accuracy is 0.334 m, 0.196 m, 0.318 m and 0.091 m when adopting four forms of IASM respectively and sequentially. The experiment results show that the positioning accuracy can be superiorly improved from 1.187 m in EWSM scheme to 0.991 m based on IASM in piecewise cosine form. It's obvious that the novel IASM can perform better than conventional EWSM in terms of positioning accuracy according to testing results.

1. Introduction

Underwater positioning technology is widely used in different kinds of marine activities, including fundamental research such as global geodesy [1–6], plate tectonics [7–11] and practical applications such as seismic risk evaluation [12], off-shore exploration [13], oceanographic survey [14,15] and marine engineering [16]. High precision and reliability of underwater positioning are required with the development of marine activities. GPS/Acoustic (GPS/A) technique has recently received a great deal of attention for marine applications. The proposition of GPS/A technique was firstly proposed and demonstrated for seafloor geodesy [17,18]. The GPS/A technique consists of two components: one is monitoring the positioning of an acoustic transponder at the sea-surface through the kinematic GPS method and the other is acoustic ranging between the transducer and transponders on the seafloor [5,19,20].

As for the acoustic data processing, the correct definition of function model and establishment of stochastic model are required to obtain high-quality estimation results of underwater static object positions. The functional model, describing the mathematical relation between the observations and the unknown parameters, is usually well known in the geometric intersection positioning [21–24]. A more perfect function model using difference technique was firstly proposed for the purpose of reducing parts of system errors and spatial correlation errors [4]. The stochastic model for underwater static objects positioning, expressing the statistical characteristics of the acoustic observations by means of a covariance matrix, however, has not received considerable attention and needs to be studied further. The stochastic model is important for parameter estimation since only when the correct stochastic model is applied can one obtain minimum variance estimators of the parameters in linear model [25,26]. There is a compelling need of improved stochastic model to be used for accurate weighting. Several kinds of optimization methods such as particle swarm optimization (PSO) and convolutional neural network (CNN) has been used and yield satisfactory results in multiple fields, for instance remote sensing, intelligence transportation, medical system, to name but a few [27,28]. As for underwater positioning, we need to improve stochastic model combined with the actual positioning situation.

The characteristic of underwater acoustic observation is evidently complicated, such as the influence of instrument noise, external environment and so on, which yield results in distinct accuracy of diverse acoustic observed value [29–31]. To the author's knowledge, the equal

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https://doi.org/10.1016/j.apor.2018.05.011
Received 2 February 2018; Received in revised form 6 May 2018; Accepted 14 May 2018

weight stochastic model is commonly used at present such as in Ocean Bottom Cable (OBC) seismic exploration under the assumption of equal prior variance of each observation, which is inconsistent with the reality and restricts the positioning accuracy improvement. In this paper, combining with underwater positioning reality, the observation errors including ranging error and incorrect acoustic ray bending error are taken into account. A new kind of stochastic model with practical significance of underwater positioning is presented, which was based on the acoustic ray incidence angle. The coordinates of underwater static object are estimated by using least squares filter ultimately.

The paper is organized as follows. Section 2 provides the geometric intersection functional model for underwater acoustic positioning. Sections 3 gives a brief derivation of defective stochastic model effect and presents the refined stochastic models: acoustic ray incidence angle stochastic model (IASM). In Section 4, to investigate the performance of the developed stochastic model, simulation test and experimental test are adopted. Finally, the conclusions are drawn in Section 5.

2. Underwater positioning function model

The function model for underwater static object positioning is briefly described in this part. First, the approximate coordinates of underwater static object are calculated. Then the precise coordinates are estimated using the least squares method.

2.1. Calculation of approximate coordinates for underwater static object

The transducer at the bottom of survey ship is at positions $X_i(x_i, y_i, z_i)$ at time t_i. The ship positioning can be estimated by the kinematic PPP technology. The PPP estimation accuracy was 2 to 3 cm horizontally and 3 to 4 cm vertically [32–34]. We assume that there is a single transponder on the seafloor, whose position $X_t^0(x_t^0, y_t^0, z_t^0)$ is to be determined from kinematic GPS and underwater acoustic ranging measurements ρ_i.

The relation of approximate coordinates of the transponder and geometrical distance between transducer and transponder can be depicted as Eq. (1):

$$S(X_t^0, X_i) = \sqrt{(x_t - x_i)^2 + (y_t - y_i)^2 + (z_t - z_i)^2} \tag{1}$$

where $S(X_t^0, X_i)$ is the geometrical distance at the epoch i.For the case where there are more than four measurements ($n \geq 4$), a linear problem can be formed involving $n-1$ normal equations that can be solved directly for the solution. A linear set of equations can be created by forming a set of $(n-1)$ linearly independent differences between each of these measurements. For example, one possible set of linearly independent differences would be

$$S(X_t^0, X_i)^2 - d(X_t^0, X_{i+1})^2 = \|X_t^0 - X_i\|^2 - \|X_t^0 - X_{i+1}\|^2 \tag{2}$$

It can be expressed in matrix equation as Eq. (3):

$$HX_t^0 = Y \tag{3}$$

where $H = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_2 & y_3 - y_2 & z_3 - z_2 \\ \vdots & \vdots & \vdots \\ x_n - x_{n-1} & y_n - y_{n-1} & z_n - z_{n-1} \end{bmatrix}$, $X_t^0 = \begin{bmatrix} x_t^0 \\ y_t^0 \\ z_t^0 \end{bmatrix}$,

$$Y = \begin{bmatrix} [d(X_t^0, X_1)^2 - d(X_t^0, X_2)^2 + (x_2^2 + y_2^2 + z_2^2) - (x_1^2 + y_1^2 + z_1^2)]^{\frac{1}{2}} \\ [d(X_t^0, X_2)^2 - d(X_t^0, X_3)^2 + (x_3^2 + y_3^2 + z_3^2) - (x_2^2 + y_2^2 + z_2^2)]^{\frac{1}{2}} \\ \vdots \\ [d(X_t^0, X_{n-1})^2 - d(X_t^0, X_n)^2 + (x_n^2 + y_n^2 + z_n^2) - (x_{n-1}^2 + y_{n-1}^2 + z_{n-1}^2)]^{\frac{1}{2}} \end{bmatrix}$$

During the actual calculation process, the geometrical distance $d(X_t^0, X_i)$ is replaced by the approximate range $\tilde{\rho}_i$, which can be calculated with the travel time and measured sound speed profile (SSP). Conceptually, the solution involving only four acoustic measurements has a straight forward geometrical interpretation. More observations are required to improve approximate coordinate accuracy of the seafloor transponder.

2.2. Geometric intersection positioning function model

The observation equation can be depicted as [4]

$$\rho_i = f(X_t, X_i) + \delta\rho_{vi} + \delta\rho_{ti} + \varepsilon_i \tag{4}$$

where $f(X_t, X_i) = \|X_t - X_i\|^2$ is the secondary module of $X_t - X_i$ and stands for the geometrical distance between transducer and transponder; $\delta\rho_{vi}$ is the ranging error caused by sound velocity error, namely the equivalent SVP (sound velocity profile) error; $\delta\rho_{ti}$ is the ranging error caused by time delay error, namely the equivalent time delay error; ε_i is the random error.

Eq. (4) can be linearized by means of Taylor series expansion method as Eq. (5):

$$\rho_i - f(X_t^0, X_i) = a_i dX + \delta\rho_{vi} + \delta\rho_{ti} + \varepsilon_i \tag{5}$$

where X_t^0 is the approximate coordinates of transponder, which is determined as mentioned above; dX is the coordinate corrections of the transponder; $X_t = X_t^0 + dX$,; a_i is the first partial derivatives of distance vector two norm $f(X_t - X_i)$ to the transponder coordinates.

If multiple distances are measured between the transducer and transponder, the matrix equation can be constructed as Eq. (6):

$$l = AdX + \varepsilon_l \tag{6}$$

where $l = \begin{bmatrix} \rho_1 - f_1^0 \\ \rho_2 - f_2^0 \\ \rho_3 - f_3^0 \\ \vdots \\ \rho_n - f_n^0 \end{bmatrix}$, $A = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x} & \frac{\partial f_n}{\partial y} & \frac{\partial f_n}{\partial z} \end{bmatrix}$, $\varepsilon_l = \begin{bmatrix} \delta\rho_{v1} + \delta\rho_{t1} + \varepsilon_1 \\ \delta\rho_{v2} + \delta\rho_{t2} + \varepsilon_2 \\ \delta\rho_{v3} + \delta\rho_{t3} + \varepsilon_3 \\ \vdots \\ \delta\rho_{vn} + \delta\rho_{tn} + \varepsilon_n \end{bmatrix}$

$$\frac{\partial f_i}{\partial x} = \frac{x_o^0 - x_i}{f_i^0}, \quad \frac{\partial f_i}{\partial y} = \frac{y_o^0 - y_i}{f_i^0}, \quad \frac{\partial f_i}{\partial z} = \frac{z_o^0 - z_i}{f_i^0}$$

$$f(X_t, X_i) = f_i, \; f(X_t^0, X_i) = \sqrt{(x_i - x_t^0)^2 + (y_i - y_t^0)^2 + (z_i - z_t^0)^2} = f_i^0$$

where l is constant term; A is the coefficient matrix of observation equation; ε_l is comprehensive error term including random error and the system error related to the sound velocity and travel time of acoustic signals.

The error equation can be depicted as Eq.7 on the premise taking no systematic error into consideration:

$$V = AdX - l \tag{7}$$

The normal equation is expressed as Eq. (8):

$$A^T PAdX - A^T Pl = 0 \tag{8}$$

3. Inaccurate stochastic model effect and incidence angle stochastic model

3.1. Inaccurate stochastic model effect analysis

This section will give the theoretical derivation and proof about the positioning effect of defective stochastic model from the aspect of mechanism.

Stochastic model can be uniformly expressed as Eq. (9):

$$D(l) = \sigma_0^2 Q = \sigma_0^2 P^{-1} \tag{9}$$

Where $D(l)$ is the Variance-covariance matrix of the underwater acoustic measurements; Q is cofactor matrix; P is the weight matrix; σ_0^2 is the variance of unit weight. We replace the parameter dX by X for