Applied Ocean Research 76 (2018) 61–70

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

Dynamics of surface and internal long waves generated by atmospheric pressure disturbances

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ARTICLE INFO

Keywords:
MCC type mathematical model
Two-layer fluid system
Atmospheric pressure disturbance
Internal wave
Froude number
Proudman resonance

ABSTRACT

Different from non-dispersive wave models such as the shallow water equations, MCC (Miyata, Choi, and Camassa) type strongly nonlinear dispersive wave model can describe not only the dynamics of surface waves, but also those of internal waves. The MCC model for a two-layer fluid system is applied to study dynamics of both surface waves and internal waves. The resonance mechanisms with atmospheric pressure disturbances are numerically studied not only for surface waves, so-called Proudman resonance, but also for internal waves. Baroclinic internal waves are generated when the speed of the atmospheric pressure disturbance is comparable to the critical linear internal wave speed. We compare the amplitudes of generated internal waves to the resonance theory for surface waves. Although most baroclinic internal wave phenomena can be explained by this linear theory, the relative magnitudes of amplitudes for forced and free waves do not agree with the theory. In addition, sequentially generated surface waves with resonance are observed when the radius of the atmospheric pressure disturbance is small. Finally, the dynamics of both surface and internal waves over non-uniform bottom topography are discussed in terms of Froude numbers to understand how they propagate and interact with each other when the Proudman resonance condition is satisfied while they travel.

1. Introduction

In many recent studies, water waves generated by moving atmospheric pressure have been considered as one of the main causes of coastal disasters. Storm surges commonly occur when a huge size of low pressure moves to a coastline. Even small atmospheric pressure disturbances can generate ocean wave, which may cause a costal disaster when interacting with non-uniform bottom topography or coastal shape. This phenomenon has been observed in many areas, e.g. Argentina [1], the Mediterranean Sea [2], the Adriatic coast [3,4], the East China Sea [5], the Louisiana shelf [6], the Balearic port of Ciutadella [7], and the west coast of Korea [8–10]. These waves are called 'meteorological tsunami' or 'meteotsunami', since they are tsunami-like waves generated by different sources. These waves can produce destructive events, even though the overall effect of the wave is much less than the seismic tsunami. The Proudman resonance [11–14] is a key mechanism in the generation of meteotsunami. When generated waves interact with coastline geometry, or when they are amplified by non-uniform bottom topography, a disaster may occur at the coast.

Proudman resonance occurs when the pressure disturbance moves at a speed similar to the linear long wave speed. Experimentally, it can be usually explained by some records from a local tidal station, by comparing the speed of theoretical linear waves near the accident area and estimating the speed of the pressure disturbances [1,3,5], when a large amplitude wave hits the coast. Shallow water equations (SWE) – or linear shallow water equations (LSWE) – have been used for mathematical models to understand Proudman resonance. Vilibić [14] investigated abnormally large waves, due to Proudman resonance with SWE, by simple numerical experiments. Vilibić et al. [15,16] and Dragani [17] also used SWE and carried out numerical experiments to analyze amplified sea level oscillations induced by moving atmospheric pressure disturbances. The effects of tides [10] and winds [4,18] are considered with SWE to understand meteotsunamis. Niu and Zhou [19] numerically investigated wave patterns produced by an atmospheric pressure disturbance, with different Froude numbers, using SWE. Choi and Seo [20] numerically studied wave run-ups by moving atmospheric pressure over a sloped beach with SWE, and compared the maximum run-up in terms of various Froude numbers. Besides SWE, a variable coefficient Korteweg-de Vries equation (vKdV) has also been used to explain lone soliton with an undular bore that occurred on the Atchafalaya shelf, Louisiana [6].

On the other hand, Proudman resonance can occur for internal

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https://doi.org/10.1016/j.apor.2018.04.010
Received 20 April 2017; Received in revised form 14 March 2018; Accepted 20 April 2018

waves in the ocean. We expect that the basic mechanism for amplifying internal waves is not that different from amplifying surface waves, since surface waves can be thought of as internal waves with greater density differences. However, internal waves are different to surface waves in their speeds and amplitudes. In general, the propagation speeds of internal solitary waves are slower and their amplitudes are larger. Thus, internal waves are usually considered as fully-nonlinear large amplitude waves existing in the pycnocline. In recent decades, many observations of internal waves have been reported due to the development of remote-sensing technology. Internal waves are known to play an important role in the transportation of energy and biological process through dissipation or mixing mechanisms. An overview of internal solitary waves and their mathematical models can be seen in [21,22].

Weakly nonlinear mathematical models – the KdV equation and its variations – are used for understanding the dynamics of internal waves, even though internal waves are highly nonlinear. It turns out that weakly nonlinear models can capture some characteristics of highly nonlinear waves. Thus, they can be considered as phenomenological models for internal waves, since it is beyond the formal validity range of the models [21]. Other than weakly nonlinear models, Miyata [23], Choi and Camassa [24] suggest a strongly nonlinear two-layer model, called MCC model, from coupled incompressible Euler equations under the long wave assumption. It removes the traditional weakly nonlinear assumption to better understand the dynamics of large amplitude internal waves. The MCC model is asymptotically the same as KdV equation and the Intermediate Long Wave equation (ILW) when additional assumptions are applied [24]. It is known that the solitary wave solutions of two-layer MCC model are well suited with experimental data and the solutions of Euler equations [25]. In addition, a multi-layer model is similarly derived in Choi [26] under the same assumption, which we call the MCC-type model. One of drawbacks of a multi-layer model is the instability of the solution, since Kelvin-Helmholtz type instabilities are common due to the small density differences [27]. The instability of internal waves in the MCC model [27], the dynamics of mode-1 and mode-2 internal waves with a three-layer fluid system [28], and the dynamics of internal waves with a multi-layer model [29] have been investigated through careful numerical treatments. However, we note that most of works on MCC model are considered with the effect of the bottom topography only.

We study the dynamics of surface waves and internal waves using a MCC-type model, including the atmospheric pressure disturbance term. In particular, we use a two-layer MCC model with non-rigid top and bottom boundaries, which we call the MCC2O model. We consider various atmospheric pressure disturbance sizes for considering the storm as well as meteotsunami events. For validation of the model and the numerical study, we compare some results with SWE, LSWE, and two-layer SWE. The Proudman resonance, for not only surface long waves but also internal waves, is discussed. Since Froude numbers for surface waves and internal waves, respectively, are key parameters, we numerically study the dynamics of ocean waves with various Froude numbers and compare the result with the linear theory. We also consider a case with relatively small radius pressure disturbances to see the effect of high order approximated terms in the model. We find a mechanism of serially generating nonlinear surface waves with resonance. Finally, we study the dynamics of waves propagating over non-uniform bottom topography. This article is organized as follows. The MCC model is described briefly in Chapter 2. (Note that detailed derivation can be found in [23,24,26]). Numerical methods for SWE, LSWE, two-layer SWE, and MCC model are also shortly described in the same chapter. Numerical results with moving atmospheric pressure disturbances and non-uniform bottom topography are discussed in Chapter 3. Dynamics of surface waves are considered in the first part and internal waves are considered in the last part of the chapter. Finally, concluding remarks are in Chapter 4.

2. Method

2.1. Governing equations

A two-layer fluid system is considered with non-rigid top boundary and non-uniform bottom topography for the MCC2O model. The subscript $i = 1, 2$ is used for the upper and lower layer, respectively. Let h_i be the undisturbed thickness and ρ_i be the density of each layer. We assume that $\rho_1 < \rho_2$ for stable stratification. Let $z = \zeta^{(s)}(x, t)$ be the equation for the top boundary – surface wave and $z = \zeta^{(i)}(x, t)$ be the equation for the interface – internal wave. Then the MCC2O model, derived from coupled incompressible Euler equations under the long wave assumption, can be written by the thickness of the layer, η_i and the averaged horizontal velocity of the layer, u_i as follows. From the mass conservation,

$$\eta_{i,t} + (\eta_i u_i)_x = 0, \tag{1}$$

where $\eta_1(x, t) = \zeta^{(s)}(x, t) - \zeta^{(i)}(x, t)$ and $\eta_2(x, t) = \zeta^{(i)}(x, t) + h_2 - b(x)$. Note that $z = b(x)$ is the equation for the bottom topography. With a given atmospheric pressure equation, $P_a(x, t)$, the evolution equations for each averaged horizontal velocity u_1, u_2 can be approximated as

$$u_{1,t} + u_1 u_{1,x} + g\zeta_x^{(s)} = -\frac{P_{a,x}}{\rho_1} + \frac{1}{\eta_1}\left(\frac{1}{3}\eta_1^3 G_1\right)_x - \frac{1}{\eta_1}\left(\frac{1}{2}\eta_1^2 K\right)_x + \left(\frac{1}{2}\eta_1 G_1 - K\right)\zeta_x^{(i)}, \tag{2}$$

$$u_{2,t} + u_2 u_{2,x} + g\zeta_x^{(i)} = -\frac{P_x}{\rho_2} + \frac{1}{\eta_2}\left(\frac{1}{3}\eta_2^3 G_2\right)_x - \frac{1}{\eta_2}\left(\frac{1}{2}\eta_2^2 B\right)_x + \left(\frac{1}{2}\eta_2 G_2 - B\right)b_x(x), \tag{3}$$

where

$$P = P_a + \rho_1\left(g\eta_1 - \frac{1}{2}G_1\eta_1^2 + K\eta_1\right),$$

$$G_i = u_{i,xt} + u_i u_{i,xx} - (u_{i,x})^2,$$

$$K = (\zeta_t^{(i)} + u_1\zeta_x^{(i)})_t + u_1(\zeta_t^{(i)} + u_1\zeta_x^{(i)})_x,$$

$$B = (\partial_t + u_2\partial_x)(u_2 b_x),$$

and g is the gravitational acceleration. A detailed derivation of the MCC model can be found in Choi and Camassa [24] and Choi [26].

We mainly use the MCC2O model and compare the results with one-layer shallow water equations (SWE1), linear shallow water equations (LSWE1), and two-layer shallow water equations (SWE2). With undisturbed water depth $h(x) = h_1 + h_2 - b(x)$, and averaged horizontal velocity $u(x)$, we note that SWE1 can be expressed as

$$H_t + (Hu)_x = 0, \tag{4}$$

$$u_t + uu_x + g\zeta_x^{(s)} = -\frac{P_{a,x}}{\rho}, \tag{5}$$

where $H(x, t) = \zeta^{(s)}(x, t) + h(x)$, and LSWE1 can be written as

$$\zeta_t^{(s)} + (hu)_x = 0, \tag{6}$$

$$u_t + g\zeta_x^{(s)} = -\frac{P_{a,x}}{\rho}. \tag{7}$$

For SWE2, the mass conservation equations are identical to Eq. (1) for each layer. Horizontal momentum equations are

$$u_{1,t} + u_1 u_{1,x} + g\zeta_x^{(s)} = -\frac{P_{a,x}}{\rho_1}, \tag{8}$$

$$u_{2,t} + u_2 u_{2,x} + g\zeta_x^{(i)} = -\frac{P_x}{\rho_2}, \tag{9}$$