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The Design Space Root Finding method for efficient risk optimization by simulation

Wellison J.S. Gomes [a], André T. Beck [b,*]

[a] Center for Optimization and Reliability in Engineering (CORE), Department of Civil Engineering, Federal University of Santa Catarina, Florianópolis, SC, Brazil
[b] Department of Structural Engineering, University of São Paulo, São Carlos, SP, Brazil

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ABSTRACT

Reliability-Based Design Optimization (RBDO) is computationally expensive due to the nested optimization and reliability loops. Several shortcuts have been proposed in the literature to solve RBDO problems. However, these shortcuts only apply when failure probability is a design constraint. When failure probabilities are incorporated in the objective function, such as in total life-cycle cost or risk optimization, no shortcuts were available to this date, to the best of the authors knowledge. In this paper, a novel method is proposed for the solution of risk optimization problems. Risk optimization allows one to address the apparently conflicting goals of safety and economy in structural design. In the conventional solution of risk optimization by Monte Carlo simulation, information concerning limit state function behavior over the design space is usually disregarded. The method proposed herein consists in finding the roots of the limit state function in the design space, for all Monte Carlo samples of random variables. The proposed method is compared to the usual method in application to one and *n*-dimensional optimization problems, considering various degrees of limit state and cost function nonlinearities. Results show that the proposed method is almost twenty times more efficient than the usual method, when applied to one-dimensional problems. Efficiency is reduced for higher dimensional problems, but the proposed method is still at least two times more efficient than the usual method for twenty design variables. As the efficiency of the proposed method for higher-dimensional problems is directly related to derivative evaluations, further investigation is necessary to improve its efficiency in application to multi-dimensional problems.

1. Introduction

In a competitive environment, engineering systems have to be designed taking into account not just their functionality, but also their expected construction and operation costs, and their capacity to generate profits. This capacity depends on the risk that construction and operation of a product or facility implies to the user, to employees, to the general public or to the environment. The capacity to generate profits can be adversely affected by the costs of failure. The performance and safety of structural systems is affected by uncertainties, or natural randomness, in the resistance of structural materials, in the loadings and in engineering models of member resistance and load effects. Uncertainty implies risk, or the possibility of undesirable structural responses.

In the context of structural optimization, many different formulations have been employed in the last years in order to find optimal designs. Among them, Deterministic Design Optimization (DDO) allows one to find the shape or configuration of a structure that is optimal in terms of mechanics, but it does not take directly into account parameter uncertainties and their effects on structural safety. A typical formulation of DDO reads:

$$\mathbf{d}^* = \arg\min\,[\mathrm{cost}(\mathbf{d}):\ \mathbf{d} \in S,\ \sigma(\mathbf{d}, \boldsymbol{\lambda}) \le \sigma_{yield}] \tag{1}$$

where $\mathbf{d}$ is a vector containing the design variables; $S=[\mathbf{d}_l, \mathbf{d}_u]$ is a vector of design constraints, with $\mathbf{d}_l$ and $\mathbf{d}_u$ the lower and upper bounds on the design variables; $\sigma(\mathbf{d}, \boldsymbol{\lambda})$represents a deterministic design constraint, such as allowable stress; and $\boldsymbol{\lambda}$ is a vector of safety coefficients, given by some design code but generally not a design variable. In Eq. (1), the cost function only includes cost (or volume) of structural materials, and sometimes manufacturing costs. Since safety is not quantified, the resulting optimal structure may compromise safety, in comparison to the original (non-

* Corresponding author.
E-mail addresses: wellison.gomes@ufsc.br (W.J.S. Gomes), atbeck@sc.usp.br (A.T. Beck).

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optimal) structure. This will generally happen as more failure modes are designed against the limit. Nowadays, it is widely recognized that DDO is not robust with respect to existing and unavoidable uncertainties in structural design.

Reliability-Based Design Optimization (RBDO) has emerged as an alternative to properly model the safety-under-uncertainty part of the problem. With RBDO, one can ensure that a minimum (and measurable) level of safety is achieved by the optimum structure, by applying a constraint on the failure probability, P_f. A typical formulation of RBDO reads:

$$\mathbf{d}^* = \arg\min\,[\mathrm{cost}(\mathbf{d}):\ \mathbf{d} \in \mathrm{S},\ P_f(\mathbf{d}, \lambda) \le P_{f_{\mathrm{admissible}}}] \tag{2}$$

where P_f represents a reliability constraint. Generally, the cost term in this formulation is the same as for DDO, that is, it does not include expected costs of failure. Thus, RBDO allows finding a structure which is optimal in a mechanical sense, and which does not compromise safety. However, results are dependent on the failure probabilities used as constraints in the analysis.

Risk Optimization (RO) increases the scope of the problem by addressing the compromising goals of economy and safety [20,5–7]. This is accomplished by quantifying the monetary consequences of failure, as well as the costs associated with construction, operation and maintenance, and by including these costs in the objective function. Thus, (reliability-based) Risk Optimization will also indirectly look for the optimum safety factors and failure probabilities:

$$\mathbf{d}^* = \arg\min\,[C_T(\mathbf{d}):\ \mathbf{d} \in \mathrm{S}] \tag{3}$$

where $C_T(\mathbf{d})$ is the total expected cost, including expected costs of failure. Since Eq. (3) has no design or reliability constraint, its solution also leads to the optimum safety factors, λ^*, or the optimum reliability constraints, $P_f^*(\mathbf{d}^*, \lambda^*)$. Expected costs of failure, for each possible failure mode of the structure, are evaluated by multiplying costs of failure by probabilities of failure. Hence, we note that in comparison to RBDO (Eq. (2)), in RO failure probability is no longer a constraint but part of the objective function. The term $C_T(\mathbf{d})$ is further detailed in Section 2.2.

A review of the literature shows that the nomenclature RBDO is indistinguishably used to describe problems where failure probabilities are design constraints [1,11–13,31,37,40,41], such as in Eq. (2), or included in the objective function [10,14,17,2,21,29,32,34], such as in Eq. (3). It should be clear that these two formulations lead to two fundamentally different problems. Risk optimization (Eq. (3)) yields an unconstrained optimization problem, characterized by the existence of multiple local minima [19,20,6]. Classical RBDO formulations, such as Eq. (2), lead to constrained optimization problems. In classical RBDO articles [1,11–13,31,37,40,41] expected costs of failure are either not considered or dismissed. In this article, we specifically address risk optimization problems.

Both RBDO and RO formulations lead to problems which are very computationally intensive to evaluate. This occurs due to the nested optimization and reliability analysis loops, which occur either with failure probability as constraints or as part of the objective function. The computational burden is particularly large when iterative numerical methods (*e.g.*, non-linear or dynamic finite element analysis) are employed in solution of the mechanical problem.

A number of approaches have been proposed in the literature in order to convert RBDO into DDO problems ([1,11–13,3,25,31,37,39–41]). When the underlying reliability problem (constraint in Eq. (2)) is solved by the First Order Reliability Method (FORM) method, nested optimization loops are obtained in the classical RBDO formulation. Since FORM is an optimization procedure itself, RBDO becomes a nested, double-loop optimization problem: the inner loop is the reliability analysis and the outer loop is the structural optimization. The coupling of these two loops leads to very high computational costs. To reduce the computational burden, several authors have proposed decoupling the structural optimization and the reliability analysis. De-coupling strategies may be divided in two types: (i) serial single loop methods and (ii) uni-level methods. The basic idea of serial single loop methods is to decouple the two loops and solve them sequentially, until some convergence criterion is achieved. On the other hand, uni-level methods employ different strategies to obtain a single loop of optimization to solve the RBDO problem. State of the art reviews of RBDO including de-coupling strategies are provided in [25,3,39].

Significantly, all de-coupling strategies mentioned above address the classical RBDO formulation (Eq. (2)), where failure probabilities are design constraints. To the best of the authors knowledge, no similar shortcuts exist for solving risk or life-cycle cost optimization problems. Thus, the computational burden associated with risk optimization remains very large.

Solution of the underlying reliability problem is a key issue in solving risk optimization problems. This is still a widely open research field, as different reliability methods found in the literature present very different computational costs and accuracies. Moreover, many new approaches have been proposed in recent years. For instance, the Stochastic Subset Simulation method proposed by Taflanidis and Beck [42] can be seen as a shortcut to solving the risk optimization problem. However, this method rapidly loses efficiency for increasing number of design variables. In the approach by Taflanidis and Beck [42], the design variables are artificially considered as uncertain and Subset Simulation is employed, in combination with a stochastic search algorithm, to solve the reliability and optimization problems simultaneously. As another example, the approach proposed by Jensen et al. [43] and applied in a risk optimization problem by Valdebenito and Schuëller [38], is very efficient even for problems involving thousands of random variables; however, the method requires many approximations, mainly when the objective function is the total expected cost.

Among the usual methods for reliability analysis, simple Monte Carlo simulation or Latin-Hypercube sampling have been employed in many applications, in combination with different optimization algorithms, mainly due to generality and ease of accuracy control. In general, accuracy of Simple Monte Carlo simulation increases with larger number of samples. However, in the risk optimization solution, one complete reliability analysis is required for each trial design. Many variants of the Monte Carlo method have been proposed in order to decrease the computational cost by decreasing the number of samples required to achieve convergence: Latin Hypercube sampling [22,33], subset simulation [4], importance sampling [15], asymptotic sampling [27]. Regardless of the sampling strategy, simulation-based methods in general only look at one information about each sample: whether it belongs to the failure domain or not. Hence, only the sign of the limit state function matters. Information about how the limit state function behaves over the design space is not computed. In this paper, it is shown that this commonly employed strategy may not be the most efficient. In this paper, a novel method is proposed, which can in principle be combined with any of the sampling schemes mentioned above. This method is based on finding, for each sample, the roots of the limit state function in the design space. This dramatically reduces the computational cost, as will be shown herein.

The core of this paper is organized in four sections. Section 2 presents the structural reliability problem and the risk optimization formulation. Section 3 describes the proposed method, focusing on its application in combination with Monte Carlo