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Validation of a global approximation for wave diffraction-radiation in deep water

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ABSTRACT

Computations of linear wave loads and second-order mean drift forces for a hemisphere and a freely floating FPSO show that global analytical approximations, given by Wu et al. in 2017, to the local flow components in the Green function and its gradient yield numerical predictions that closely agree with numerical results obtained via a highly-accurate Green function, as well as analytical results for a hemisphere. The computations reported here provide strong evidence that the global analytical approximations, valid within the entire flow region, are sufficiently accurate to compute linear and mean drift wave loads in practice.

1. Introduction

Boundary integral equations related to the linear potential flow theory of diffraction-radiation of regular (time-harmonic) water waves are routinely solved to predict wave loads and wave-induced motions of large floating structures [1–5]. An essential ingredient of that classical method is the Green function that satisfies the linearized boundary condition at the free surface and the radiation condition [6]. Practical, reliable predictions of wave loads and wave-induced motions require that the Green function, and its gradient, be evaluated efficiently and with sufficient accuracy.

The Green function can be expressed as the sum of the free-space singularity (Rankine source) and a component that accounts for the free surface. This free-surface component can be further decomposed into a non-oscillatory local flow component and a wave component, which is dominant in the far field [7]. The wave component in this basic decomposition is defined in terms of special (real) functions (Struve functions $\tilde{H}_0$ and $\tilde{H}_1$ and Bessel functions J_0 and J_1) of a single variable. These functions are finite and smooth everywhere, and can be evaluated accurately and efficiently [8–11].

The non-oscillatory local flow components in the expressions for the Green function and its gradient are real functions of two variables and are defined by single integrals. Nearfield and farfield analytical approximations to these local flow components are given in [7] and used in [12]. These analytical approximations can also be used to remove the nearfield singularities and to reduce the unbounded region to a finite region, and thus make it possible to use polynomial approximations within contiguous flow regions [13,14] or table interpolation [15].

The nearfield and farfield analytical approximations to the local flow components in the Green function and its gradient are also used in [16] to obtain global analytical approximations that are valid within the entire flow region. These global approximations are less accurate than the approximations based on series expansions [12], polynomial approximations within contiguous flow regions [13,14] or table interpolation [15] previously given in the literature, but are a particularly simple and intellectually satisfying alternative. Moreover, global approximations offer a significant practical advantage because they avoid the need for "if statements", required if different approximations are used within complementary contiguous regions, and are then especially well suited for highly efficient parallel computations.

The error analysis given in [16] suggests that the global analytical approximations given in that study could be expected to be sufficiently accurate for most practical applications. However, the error analysis given in [16] is limited and insufficient. Indeed, a more solid conclusion requires computations of linear wave loads and second-order mean drift forces. Such computations are reported here for a hemisphere and a Floating Production Storage and Offloading (FPSO) unit.

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2. Green function and basic decomposition

Thus, the Green function related to diffraction-radiation of regular (time-harmonic) water waves in the lower half space $Z \le 0$ is now considered. A Cartesian coordinate system XYZ is defined. The XY plane coincides with the undisturbed free surface and the Z axis points upward. Time-harmonic flows associated with flow potentials of the form

$$\Re\left[\Phi(\mathbf{X})\,\mathrm{e}^{-\mathrm{i}\omega T}\right] \tag{1}$$

are considered, and nondimensional coordinates are defined as

$$(x, y, z) \equiv (X, Y, Z)\,\omega^2/g \tag{2}$$

Here, ω denotes the circular frequency and g is the acceleration of gravity. The Green function $G(\boldsymbol{x}, \boldsymbol{\xi})$, where $\boldsymbol{x} \equiv (x, y, z \le 0)$ and $\boldsymbol{\xi} \equiv \left(\xi, \eta, \zeta \le 0\right)$ denote a flow-field point and a source point, is expressed in [6] as

$$4\pi G = \frac{-1}{r} - \frac{1}{d} - 2\left[\int_0^\infty \frac{\mathrm{e}^{kv}}{k-1} J_0(kh)\,dk + \mathrm{i}\pi\mathrm{e}^{v} J_0(h)\right] \tag{3}$$

where h, v, r and d are defined as

$$h \equiv \sqrt{\left(x-\xi\right)^2 + (y-\eta)^2}, \quad v \equiv z + \zeta \le 0, \tag{4a}$$

$$r \equiv \sqrt{h^2 + (z-\zeta)^2}, \quad d \equiv \sqrt{h^2 + (z+\zeta)^2} = \sqrt{h^2 + v^2}, \tag{4b}$$

and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind.

The Green function G can be expressed as

$$4\pi G = -1/r - 1/d + L + W \tag{5}$$

where L represents a non-oscillatory local flow component and W is a wave component. The basic decomposition (5) of the Green function is not unique. Several alternative representations are considered in [7]. The representation (5.11) in [7] is adopted here. The wave component W in this representation is given by

$$W = 2\pi\mathrm{e}^{v}\left[\tilde{H}_0(h) - \mathrm{i}J_0(h)\right] \tag{6}$$

where $\tilde{H}_0(\cdot)$ denotes the zeroth-order Struve function. The local flow component L in (5) is given by

$$L = -\frac{4}{\pi}\int_0^{\frac{\pi}{2}} \Re\left[\mathrm{e}^{M}E_1(M)\right]\mathrm{d}\theta \quad \text{where} \quad M \equiv v + \mathrm{i}\,h\cos\theta \tag{7}$$

and $E_1(\cdot)$ denotes the complex exponential-integral function.

The gradient of the Green function is given by

$$4\pi G_z = \frac{z-\zeta}{r^3} + \frac{v}{d^3} + L_z + W \quad \text{where} \quad L_z = \frac{-1}{d} + L \tag{8a}$$

$$4\pi G_h = \frac{h}{r^3} + \frac{h}{d^3} + L_h + W_h \tag{8b}$$

$$4\pi G_x = \frac{x-\xi}{h}G_h \quad \text{and} \quad 4\pi G_y = \frac{y-\eta}{h}G_h \tag{8c}$$

The derivatives $W_h \equiv \partial W/\partial h$ and $L_h \equiv \partial L/\partial h$ are given by

$$W_h = 2\pi\mathrm{e}^{v}[2/\pi - \tilde{H}_1(h) + \mathrm{i}\,J_1(h)] \tag{9}$$

where $\tilde{H}_1(\cdot)$ and $J_1(\cdot)$ denote the first-order Struve function and the first-order Bessel function of the first kind, respectively, and

$$L_h = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \Im\left[\mathrm{e}^{M}E_1(M) - \frac{1}{M}\right]\cos\theta\,\mathrm{d}\theta \tag{10}$$

The wave components W and W_h defined by (6) and (9) are indefinitely differentiable. Practical polynomial approximations to the Struve functions $\tilde{H}_0$ and $\tilde{H}_1$ and the Bessel functions J_0 and J_1 are given in e.g. [8,9], and the wave component W and its derivative W_h in (5) and (8) can then be evaluated efficiently and very simply.

The local flow components $L(h, v)$ and $L_h(h, v)$ defined by (7) and (10) vanish as $d \to \infty$ and are singular as $d \to 0$. In particular, in the limit $d \to 0$, [7,16] show that one has

$$L \sim 2\left(\log\frac{d-v}{2} + \gamma\right) \quad \text{and} \quad L_h \sim \frac{2h}{d-v}\left(\frac{1}{d} + 1\right) - 4 \tag{11}$$

where $\gamma \approx 0.577$ is Euler's constant. The functions $L(h, v)$ and $L_h(h, v)$ are depicted in Figs. 8 and 9 of [16].

3. Global approximations to the local components L and L_h

The local flow components L and L_h can be evaluated, very simply and efficiently, via the global analytical approximations obtained in [16] by extending the nearfield and farfield approximations given in [7]. These practical global approximations, valid within the entire flow region ($0 \le h, v \le 0$), are now considered. The three parameters

$$0 \le \alpha \equiv \frac{-v}{d} \le 1, \quad 0 \le \beta \equiv \frac{h}{d} \le 1, \quad 0 \le \rho \equiv \frac{d}{1+d} < 1 \tag{12}$$

are used.

The local flow component L defined by the integral (7) is approximated in [16] as

$$L \approx L^a \equiv 2P/(1+d^3) + 2\rho(1-\rho)^3 R \,\text{where} \tag{13a}$$

$$P \equiv \mathrm{e}^{v}\left(\log\frac{d-v}{2} + \gamma - 2d^2\right) + d^2 - v\,\text{and} \tag{13b}$$

$$R \equiv \left(1-\beta\right)A - \beta B - \frac{\alpha C}{1 + 6\alpha\rho(1-\rho)} + \beta\left(1-\beta\right)D \tag{13c}$$

A, B, C and D in (13c) are polynomials in ρ defined as

$$\begin{aligned} A \equiv\; & +1.21 - 13.328\rho + 215.896\rho^2 - 1763.96\rho^3 + 8418.94\rho^4 \\ & -24314.21\rho^5 + 42002.57\rho^6 - 41592.9\rho^7 \\ & +21859\rho^8 - 4838.6\rho^9 \end{aligned} \tag{14a}$$

$$\begin{aligned} B \equiv\; & +0.938 + 5.373\rho - 67.92\rho^2 + 796.534\rho^3 - 4780.77\rho^4 \\ & +17137.74\rho^5 - 36618.81\rho^6 + 44894.06\rho^7 \\ & -29030.24\rho^8 + 7671.22\rho^9 \end{aligned} \tag{14b}$$

$$\begin{aligned} C \equiv\; & +1.268 - 9.747\rho + 209.653\rho^2 - 1397.89\rho^3 + 5155.67\rho^4 \\ & -9844.35\rho^5 + 9136.4\rho^6 - 3272.62\rho^7 \end{aligned} \tag{14c}$$

$$\begin{aligned} D \equiv\; & +0.632 - 40.97\rho + 667.16\rho^2 - 6072.07\rho^3 + 31127.39\rho^4 \\ & -96293.05\rho^5 + 181856.75\rho^6 - 205690.43\rho^7 \\ & +128170.2\rho^8 - 33744.6\rho^9 \end{aligned} \tag{14d}$$

The local flow velocity component L_h defined by (10) is similarly approximated in [16] as

$$L_h \approx L_h^a \equiv 2P_*/(1+d^3) - 4Q_* + 2\rho(1-\rho)^3 R_* \tag{15a}$$

$$\text{where} \quad P_* \equiv (\beta + h)/(d-v) - 2\beta + 2d\mathrm{e}^{v} - h, \tag{15b}$$

$$Q_* \equiv \mathrm{e}^{-d}\left(1-\beta\right)\left(1 + \frac{d}{1+d^3}\right) \quad \text{and} \tag{15c}$$