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Maximum entropy modeling of discrete uncertain properties with application to friction

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ABSTRACT

The first part of the present investigation focuses on the formulation of a novel stochastic model of uncertain properties of media, homogenous in the mean, which are represented as stationary processes. In keeping with standard spatial discretization methods (e.g., finite elements), the process is discrete. It is further required to exhibit a specified mean, standard deviation, and a global measure of correlation, i.e., correlation length. The specification of the random process is completed by imposing that it yields a maximum of the entropy. If no constraint on the sign of the process exists, the maximum entropy is achieved for a Gaussian process the autocovariance of which is constructed. The case of a positive process is considered next and an algorithm is formulated to simulate the non-Gaussian process yielding the maximum entropy.

In the second part of the paper, this non-Gaussian model is used to represent the uncertain friction coefficient in a simple, lumped mass model of an elastic structure resting on a frictional support. The dynamic response of this uncertain system to a random excitation at its end is studied, focusing in particular on the occurrence of slip and stick.

1. Introduction

The modeling of uncertain geometric and/or material properties as random variables, stochastic processes, or random fields has received significant attention in the last 2 decades, e.g., see [4,7,8,14] for discussion and some methods review. A key challenge in using such models in practical situations is the often dramatic lack of information available on the uncertainty. More specifically, the mean value of the geometric/material property considered is generally believed to be known from past experience and/or available data. However, even the level of uncertainty, e.g., standard deviation, is often less clearly known and may in fact be considered as a variable in a parametric study. Sometimes, an upper and/or lower bound may also be known because of an acceptance/rejection test carried out on all samples. However, more detailed information is very often not available.

This limited data does not permit the analyst to build a specific probabilistic model, he/she is then forced to make additional assumptions. This perspective has led [11,12] to propose that the stochastic description of the uncertainty model be *derived* by *maximizing the statistical entropy* under constraints representing the true knowledge on the stochastic model. The maximization of the entropy leads to a maximum spread of the uncertainty (consistently with the constraints) in the tail of the distribution and thus to the consideration of "wide-spread" uncertainty that provides a good perspective on the effects of variations of the system properties, even those that are "far" from the mean. Thus, this approach, also referred to as the nonparametric stochastic modeling approach, requires only partial knowledge of the system uncertainty complemented by the single assumption of maximization of entropy.

In its original formulation [11,12], the nonparametric stochastic modeling approach was used for characterization of the mass, damping, and stiffness matrices of reduced order/modal models, not to the detailed modeling of any specific property such as mass density or Young's modulus. Such a characterization was carried out in later extension [13] to matrix-valued fields focusing in particular on the modeling of the elasticity tensor of random media.

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The present effort complement this work by addressing the modeling of scalar random properties, of variable or constant sign and homogenous in the mean, of media as stationary processes. In keeping with standard spatial discretization methods (e.g., finite elements), the process is discrete. The proposed modeling relies on the specification of only the mean and standard deviation of the property as well as on a global measure of the correlation, i.e., a correlation length. The maximization of the entropy then provides the description of the process consistent with this prescribed information.

The above stochastic modeling technique will be exemplified here on a property seldom considered and yet exhibiting well known uncertainty, i.e., friction. When two deformable bodies are in extended contact with each other, as in joints, turbomachinery blade friction dampers, brakes, etc., the coefficient of friction between them must be defined over the contact zone, i.e., as a spatially varying property which governs the occurrence of stick, microslip, or macroslip (e.g., see [1,3,9,10], and references therein). After an appropriate spatial discretization of the contact zone, it becomes then necessary to specify the coefficient of friction at a set of discrete locations. The uncertainty in the values of this coefficient resulting from unknown spatial variations of roughness, temperature, composition, etc. then calls for the stochastic modeling considered in the first part of the paper. To demonstrate this application and presents a first perspective on this problem, a simple dynamic model of this contact problem is adopted here as a cascade of 5 oscillators with Jenkins friction elements. The effects of uncertainty in the coefficients of friction on the dynamic response of this system is then studied.

2. Maximum entropy discrete process

2.1. General derivation

Let X_n denote a discrete stationary process indexed by the set of all the negative and positive integers. Further, let $n \in I = \{ l, l+1, \dots, u\}$ and define the random vector $\underline{X} = [X_l\, X_{l+1} \dots X_u]^T$ where ^{T} denotes the operation of matrix/vector transposition. Then, the entropy S of $\underline{X}$ is defined as

$$S \equiv -E\left[\ln\left(p_{\underline{X}}(\underline{x})\right)\right] = -\int_{\Omega} \ln\left(p_{\underline{X}}(\underline{x})\right) p_{\underline{X}}(\underline{x})\, d\underline{x} \tag{1}$$

where $E[.]$ denotes the operation of mathematical expectation and Ω is the domain of support of the values of the process. If no signature constraint is enforced, both positive and negative values of the process are allowed and thus

$$\Omega = \left\{ (x_l, x_{l+1}, \dots, x_u) \in (-\infty, \infty) \times (-\infty, \infty) \dots \times (-\infty, \infty) \right\}\cdot \tag{2}$$

If a positive sign of the process is required,

$$\Omega = \left\{ (x_l, x_{l+1}, \dots, x_u) \in [0, \infty) \times [0, \infty) \dots \times [0, \infty) \right\}\cdot \tag{3}$$

In Eq. (1), $p_{\underline{X}}(\underline{x})$ denotes the probability density function of the random vector $\underline{X}$ evaluated at a realization point $\underline{x}$. Since X_n is stationary, its joint probability density functions satisfy the usual independence under a uniform shift along I, e.g.,

$$p_{X_n}(x) = p_{X_{n+p}}(x)$$

and

$$p_{X_n X_m}(x, y) = p_{X_{n+p} X_{m+p}}(x, y) \tag{4}$$

for every n, m, $n+p$, and $m+p$ belonging to I.

It is desired here to determine the probability density function $p_{\underline{X}}(\underline{x})$ which maximizes the entropy, Eq. (1), under the constraints that:

(i) the total probability is one, i.e.,

$$\int_{\Omega} p_{\underline{X}}(\underline{x})\, d\underline{x} = 1 \tag{5}$$

(ii) the mean and variance are given and constant (since the process is stationary)

$$\int_{\Omega} x_n\, p_{\underline{X}}(\underline{x})\, d\underline{x} = \mu_X \;\; n \in I \tag{6}$$

$$\int_{\Omega} (x_n - \mu_X)^2\, p_{\underline{X}}(\underline{x})\, d\underline{x} = \sigma_X^2 \text{ for } n \in I \tag{7}$$

(iii) a correlation length is given. Two such measures are [6,13]

$$L_0 = \sum_{m=1}^{\infty} |K_{XX}(m)|/K_{XX}(0) \tag{8}$$

and

$$L_1 = \sum_{m=0}^{\infty} m\; |K_{XX}(m)| / \sum_{m=0}^{\infty} |K_{XX}(m)| \tag{9}$$

where $K_{XX}(m)$ is the stationary autocovariance function. It is defined as

$$K_{XX}(m) = \Gamma_{XX}(n, n+m) \text{ for any } n \in I \tag{10a}$$

where

$$\Gamma_{XX}(n, n+m) = E\left[(X_n - \mu_X)(X_{n+m} - \mu_X)\right] = \int_{\Omega} (x_n - \mu_X)(x_{n+m} - \mu_X)\, p_{\underline{X}}(\underline{x})\, d\underline{x} \tag{10b}$$

The series involved in Eq. (8) and (9) will be truncated to finite sums for m=0 to $m_{\max}$ and thus the constraints can be written as

$$\sum_{m=0}^{m_{\max}} a_m\, s_m\, \Gamma_{XX}(n, n+m) = 0 \text{ for any } n \tag{11}$$

where

$$s_m = \operatorname{sgn}\left(K_{XX}(m)\right) \tag{12}$$

$$a_m = \left(1 + L_0\right)\, \delta_{m0} - 1 \;\; m \in \left[0, m_{\max}\right], \tag{13}$$

with δ_{ij} the Kronecker symbol, for the correlation length L_0 while for L_1

$$a_m = L_1 - m \;\; m \in \left[0, m_{\max}\right]\cdot \tag{14}$$

For the optimization of the entropy, Eqs (5)–(7), (11) can be written in the generic form

$$\Xi_{in} \equiv \int_{\Omega} f_{in}(\underline{x})\, p_{\underline{X}}(\underline{x})\, d\underline{x} - C_i = 1, 2, 3, 4 \tag{15}$$

where

$$f_{1n}(x_n) = x_n - \mu_X \tag{16}$$

$$f_{2n}(\underline{x}) = \sum_{m=0}^{m_{\max}} a_m\, s_m\, (x_n - \mu_X)(x_{n+m} - \mu_X) \tag{17}$$

$$f_{3n}(\underline{x}) = (x_n - \mu_X)^2 \text{ and } f_4 = 1 \tag{18,19}$$

with $C_1 = C_2 = 0$, $C_3 = \sigma_X^2$, and $C_4 = 1$.

Then, the maximization of the entropy, Eq. (1) under the constraints of Eqs (15)–(19) can be accomplished in the Lagrange multiplier framework and yields