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Reliability assessment of structural systems with interval uncertainties under spectrum-compatible seismic excitations

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ABSTRACT

The paper deals with reliability assessment of linear structures with interval parameters subjected to seismic excitations modeled as stationary spectrum-compatible Gaussian random processes. Under the Vanmarcke assumption that the up-crossings of a specified threshold occur in clumps, an efficient procedure for the evaluation of the bounds of the *interval reliability function* of the generic response process is presented. The key idea is to view the *interval reliability function* as depending on the first three *interval spectral moments* of the structural response rather than on the interval structural parameters. Then, the lower and upper bounds of the *interval reliability function* are obtained by properly combining the bounds of the *interval spectral moments*. Such bounds are evaluated in approximate form by applying an approach recently proposed by the authors. Finally, based on the knowledge of the *interval reliability function*, a method for evaluating the *interval survival probability* of a structural element with imprecise strength *probability density function* is developed.

1. Introduction

Reliability assessment of randomly excited structures with uncertain parameters is a topic of great interest for design purposes. Indeed, environmental loads, such as earthquake ground motion, sea waves or gusty winds, are commonly modeled as random processes. On the other hand, the performance and reliability of structural systems may be seriously affected by uncertainties inherent in material and/or geometric properties [1].

Several studies have been carried out within a probabilistic framework (see e.g., [2,3]) modeling the excitations as stationary or non-stationary random processes, the uncertain parameters as random variables or random fields and then focusing on the estimation of the statistics of the success probability (or reliability). Much less attention has been devoted to reliability analysis of structures with uncertain parameters handled by non-probabilistic approaches, such as convex models, fuzzy set theory or interval model [4].

After the pioneering study by Ben-Haim [5], who first introduced a non-probabilistic concept of reliability, the application of non-traditional uncertainty models to structural safety assessment has increasingly spread (see e.g., [6–14]). Among these approaches, the interval model, originally developed on the basis of the *interval analysis* [15], turns out to be very useful for handling non-deterministic properties described by range information only. To the best of the authors' knowledge, available contributions in the field of non-probabilistic reliability analysis mainly focus on static problems, whereas only a few recent papers deal with the reliability assessment of randomly excited structures (see e.g., [16–18]).

This study presents an efficient mixed probabilistic and non-probabilistic procedure for reliability analysis of linear structures with r_K uncertain-but-bounded structural parameters subjected to seismic excitations modeled as stationary Gaussian spectrum-compatible random processes. In this context, the *cumulative distribution function* (*CDF*) of the *extreme value* random response process coincides with the *reliability function*, i.e. the time-dependent probability of success [19–21]. In this paper, the *reliability function* is evaluated under the Vanmarcke [22] assumption that up-crossings of a specified threshold occur in clumps when narrow band processes are involved. Under this hypothesis, the *reliability function* depends on the first three *spectral moments* (*SMs*) of the selected structural response process. Since the interval model is adopted to describe the uncertain structural parameters,

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the statistics of the random response along with the *reliability function* turn out to have an interval nature. Within this context, structural reliability assessment requires the evaluation of the bounds of the *interval reliability function* of the selected structural response process. To this aim, it is more convenient, from a computational point of view, to consider the *interval reliability function* as depending on the first three *interval SMs* of the response process rather than on the r_K interval structural parameters. So operating, for a given barrier level, the bounds of the *interval reliability function* are determined as the minimum and maximum among the values of the *reliability function* corresponding to all possible combinations of the endpoints of the *interval SMs*, say 2^3. The proposed approach thus allows a drastic reduction of the computational effort required by the classical combinatorial procedure, known as *vertex method*. Indeed, for a structure with r_K interval parameters, the *vertex method* involves 2^{r_K} reliability analyses, so that it becomes prohibitive as the number of uncertain parameters increases.

Various approaches are available in the literature to derive the *power spectral density* (*PSD*) function of ground acceleration compatible with the target spectrum (see e.g., [23–27]). Among these approaches, the method proposed by Cacciola et al. [27] is here adopted. The main advantage of this method lies in the use of a handy recursive formula for generating a spectrum-consistent *PSD* function.

It is recalled that one of the main purposes of structural engineering is to provide a measure of the risk of a structural system in terms of *probability of failure* and a measure of the success in terms of *probability of success* or *survival probability* [28]. In the paper, a method is presented to evaluate the *interval survival probability* of a selected structural element based on the knowledge of its *interval reliability function* and of the *probability density function* (*PDF*) of the strength of the constituent material. On account of the interval nature of material properties, an imprecise *PDF* [29], depending on interval distribution parameters, is assumed for the strength. Then, the bounds of the *interval survival probability* (or *interval failure probability*) are derived by applying standard probabilistic concepts.

The main steps required by the proposed approach for interval reliability analysis may be summarized as follows: (i) to determine the bounds of the *interval SMs* of a selected structural response process in approximate form by applying the *improved interval analysis via extra unitary interval* (*IIA via EUI*) [30] combined with the *Interval Rational Series Expansion* (*IRSE*) [31,32]; (ii) to evaluate the *reliability function* corresponding to the 2^3 combinations of the bounds of the *interval SMs* of the structural response and then seek the minimum and maximum for a given barrier level; (iii) to determine the *interval survival probability* of a selected structural element given the imprecise strength *PDF*.

In the numerical application section, a spatial frame with interval Young's moduli under spectrum-compatible seismic excitation is analyzed. For validation purpose, first the bounds of the *interval reliability function* obtained by applying the proposed procedure are compared with the ones provided by the *vertex method*; then, the *interval survival probability* is determined for two selected columns of the spatial frame.

2. Preliminary concepts

2.1. The role of spectral moments in structural reliability analysis

Structural systems are conceived and designed to survive natural actions. If the excitations are modeled as random processes, the dynamic response of interest is described by a random process too and structural safety needs to be evaluated in a probabilistic sense. Among the models of failure available in the literature, the simplest one, which is also the most widely used in practical analyses, is based on the assumption that a structure fails as soon as the response at a critical location exits a prescribed safe domain for the first time [19,20]. In practice, introduced the *extreme value* random process, $Y_{max}(T_S)$, of a selected structural response quantity, $Y(t)$ (e.g., strain or stress at a critical point), within a specified time interval T_S (T_S being the time observing window):

$$Y_{max}(T_S) = \max_{0 \le t \le T_S} |Y(t)| \tag{1}$$

and defined a deterministic barrier b, the *cumulative distribution function* (*CDF*) of $Y_{max}(T_S)$, $L_{Y_{max}}(b, T_S)$, coincident with the *reliability function*, i.e. the *probability of success*, can be defined as

$$L_{Y_{max}}\big(b, T_S\big) = \mathcal{P}\big[Y_{max}(T_s) \le b\big]. \tag{2}$$

In the previous equations, the symbol $|\bullet|$ denotes absolute value, while $\mathcal{P}[\bullet]$ gives the probability associated with the event into square brackets.

Unfortunately, the prediction of the *probability of success* is one of the most complicated problems in random vibration theory. The solution of this problem has not been derived in exact form, even in the simplest case of the stationary response of a single-degree-of-freedom (SDoF) linear oscillator under zero-mean Gaussian white noise [19]. Hence, a large number of approximate techniques has been proposed in the literature, which differ in generality, complexity and accuracy. In the framework of approximate methods, in the case of linear structures subjected to stationary zero-mean random processes, the *reliability function* of the structural response random process $Y(t)$ can be expressed as [22]

$$L_{Y_{max}}\big(b, T_S\big) \approx L_{Y_{max}}(b, 0)\exp\big[-T_S\,\eta_Y(b)\big] \tag{3}$$

where $\eta_Y(b)$ is the so-called *hazard function* and $L_{Y_{max}}(b, 0) \equiv \mathcal{P}[Y_{max}(0) \le b]$ is the probability of success at time $t = 0$ which, without loss of generality, is herein assumed equal to unity. For narrow band zero-mean Gaussian random processes, the *hazard function* has been derived by Vanmarcke [22] in the following form:

$$\eta_Y(b) = \frac{1}{\pi}\sqrt{\frac{\lambda_{2,Y}}{\lambda_{0,Y}}}\left[\frac{1-\exp\left(-b\,\delta_Y^{1.2}\sqrt{\dfrac{\pi}{2\lambda_{0,Y}}}\right)}{\exp\left(\dfrac{b^2}{2\lambda_{0,Y}}\right)-1}\right] \tag{4}$$

where δ_Y is the so-called bandwidth parameter of the random process $Y(t)$ [22] defined as

$$\delta_Y = \sqrt{1 - \frac{\lambda_{1,Y}^2}{\lambda_{0,Y}\lambda_{2,Y}}}. \tag{5}$$

This parameter measures the narrowness of the stochastic process $Y(t)$. In the previous equations, $\lambda_{\ell,Y}$ ($\ell = 0, 1, 2$) are the so-called *spectral moments* (*SMs*) of the stochastic process $Y(t)$ defined by Vanmarcke [33] as the geometric moments of its one-sided *power spectral density* (*PSD*) function, $G_{YY}(\omega)$, that is:

$$\lambda_{\ell,Y} = \int_0^\infty \omega^\ell G_{YY}(\omega)\mathrm{d}\omega, \quad (\ell = 0, 1, 2). \tag{6}$$

Then, according to Vanmarcke [22], the *reliability function* can be evaluated as a function of the zeroth, first- and second-order *SMs*.

2.2. Response spectrum-compatible artificial stationary earthquake accelerograms

The problem of simulating response spectrum-compatible earthquake accelerograms is addressed on a probabilistic basis