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Harmonic wavelets based excitation–response relationships for linear systems: A critical perspective

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ABSTRACT

An analytical approach based on generalized harmonic wavelets (GHWs) for determining the response evolutionary power spectral density (PSD) of single-degree-of-freedom linear structural systems subjected to non-stationary stochastic excitations is developed. Specifically, utilizing a periodized GHW transform, applying a Galerkin scheme, and relying on the orthogonality properties of the GHWs, an analytical relationship between wavelet coefficients of the system response and of the excitation is derived. It is shown that in comparison with a recently developed GHW based excitation–response relationship, which is based on a "locally stationary" stochastic process representation and employs localized in time monochromatic functions, the herein developed relationship exhibits enhanced accuracy for cases of relatively flexible systems and/or for systems with relatively low damping where the related impulse response function cannot be assumed to be short-lived. Pertinent numerical examples and Monte Carlo simulation data are included as well for demonstrating the reliability of the approach.
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1. Introduction

Due to randomness related to the system parameters/properties, and/or to the environmental loads, structural system responses exhibit random characteristics as well. Clearly, this necessitates appropriate stochastic modeling of the excitations and/or system parameters, as well as the utilization of potent tools from random vibration theory for determining the stochastic response of such systems. Nevertheless, in many cases engineering structures are subject to excitations that exhibit strong variability in both the time and the frequency domains. In this regard, well-established standard tools from random vibration theory are not capable, in general, of treating such cases adequately, and thus, developing techniques for efficient joint time–frequency system response analysis remains a sustained challenge.

Modern wavelet analysis, originated from the field of exploration geophysics in 1980s [1], is currently regarded as one of the most potent signal processing tools and has been applied in diverse fields of engineering such as dynamical systems and vibrations [2], joint time–frequency analysis [3], evolutionary power spectral density (PSD) estimation [4], structural system stochastic response determination [5], system parameter identification [6], and damage detection applications [7]; see also [8] for a comprehensive state-of-the-art review. Note that in the aforementioned applications the capabilities of the wavelet transform for joint time–frequency analysis provide with enhanced information regarding the signal's time-dependent frequency content; this is not available by merely considering the time domain only, or the frequency domain only, independent of one another.

In this regard, Basu and Gupta [9–11], Tratskas and Spanos [5], Spanos and Kougioumtzoglou [12,13] and Kong et al. [14] constitute some indicative contributions to this field. Specifically, based on the assumption that the wavelet transform of a stochastic excitation can be represented as a multiplication of two sinusoidal functions, Basu and Gupta [9] derived a relationship between the PSDs of the excitation and of the system response; this was done in conjunction with the modified Littlewood–Paley wavelet; see also [10,11]. Further, by extending the concept of the frequency response function to the joint time–frequency domain, Tratskas and Spanos [5] showed that the wavelet coefficients of the system response can be determined by applying a convolution operator to

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the wavelet coefficients of the excitation and the wavelet based impulse response function at each scale. Note, however, that the response PSD determination relied on a Monte Carlo simulation (MCS) treatment where the ensemble average of the system response wavelet coefficients needed to be considered.

Recently, based on a locally stationary representation of nonstationary stochastic processes [15] via generalized harmonic wavelets (GHWs) [16], Spanos and Kougioumtzoglou [12,13] (see also [17]) derived a GHW-based relationship between the excitation and the response PSDs. Based on certain assumptions, an important feature of the relationship is that the response PSD value at a specific time–frequency band depends on the excitation PSD value corresponding to the same time–frequency band only. In the ensuing analysis, this is referred to as the locally stationary approach and has been shown to provide with results of satisfactory accuracy given that the system under consideration is relatively stiff, and/or has relatively high damping, or in other words, its impulse response function is short-lived.

In the present paper, a novel wavelet based relationship between the excitation and the system response PSDs is derived that can be construed as a generalization of the locally stationary approach. In the herein derived relationship the assumption of local stationarity is removed, and thus, enhanced accuracy is exhibited in cases of lightly damped systems as well. To this aim, first, periodized generalized harmonic wavelets (PGHWs) are presented and utilized as an orthogonal basis for expanding the excitation and the system response functions, and ultimately, for solving the differential equation of motion. In this manner, wavelet coefficients corresponding to the deterministic system response at different scales are obtained via an analytical closed-form formula. Finally, besides obtaining the deterministic response of the system by employing a fast PGHW reconstruction algorithm, a direct excitation–response (input–output) PSD relationship is established by invoking a recently derived GHW based PSD estimation formula. Pertinent Monte Carlo simulation data are included as well demonstrating the enhanced accuracy exhibited by the proposed method as compared with the locally stationary method for a certain class of problems.

2. Mathematical formulation

Wavelet based solution of differential/integral equations has been the focus of research for over twenty years, e.g. [18]. In this regard, a wavelet-based solution of differential equations relies heavily on the determination of the “connection coefficients” representing the interaction between wavelets (or derivatives/integrals of wavelets) at different scales and translation levels. Indicatively, connection coefficients have been obtained in [19]. Further, Chen et al. [20] (see also Zhang et al. [21]) obtained the connection coefficients of Daubechies wavelets on a finite interval. As far as HWs are concerned, connection coefficients were first obtained by Cattani [22] and later employed in a collocation solution of the Fredholm integral equation of the second kind [23,24]. Further, HWs were also used in the numerical solution of Burgers equation [25].

In this section, a GHW approach is developed for solving the deterministic/stochastic differential equation of motion of a linear oscillator. In this regard, based on GHW expansions of the system response and excitation a set of algebraic equations is derived for determining the system response wavelet coefficients. The advantages of implementing such a wavelet expansion scheme relate to the sparse, in general, representation of the involved functions and differential/integral operators as well as to the existence of efficient numerical algorithms for the forward and the inverse wavelet transforms.

Nevertheless, one main difficulty in employing a GHW expansion is that GHW do not form a set of orthogonal basis functions on a finite interval. Therefore, either periodization of wavelets needs to be implemented or the wavelet basis functions at the boundaries/initial points need to be modified to obtain orthogonal wavelet bases. In the ensuing analysis, the former solution is adopted and the signal finite time interval is treated as a fundamental period of an infinite in time periodic process. For this purpose, first, the periodized generalized harmonic wavelet (PGHW) is presented. Further, PGHW connection coefficients are derived in an analytical form and utilized for obtaining a direct relationship between wavelet coefficients of the system response and of the excitation. Next, for the purpose of reconstructing the deterministic system response in the time domain from its wavelet coefficients fast forward and inverse wavelet transform algorithms based on the Fast Fourier Transform (FFT) can be utilized. Finally, a direct relationship between the excitation and the system response wavelet-defined PSDs is derived.

2.1. Periodized generalized harmonic wavelets

2.1.1. Basic formulation

The periodization of the GHW can be expressed as (e.g.[14,26])

$$\psi^{G,\mathrm{per}}_{(m_i,n_i),k}(t)=\sum_{p=-\infty}^{\infty}\psi^{G}_{(m_i,n_i),k}\left(t-pT_0\right),\tag{1}$$

where $\psi^{G}_{(m_i,n_i),k}(t)$ is the GHW attaining a representation in the time domain of the form

$$\psi^{G}_{(m_i,n_i),k}(t)=\frac{\exp\left[\mathrm{i}n\Delta\omega\left(t-\frac{kT_0}{n-m}\right)\right]-\exp\left[\mathrm{i}m\Delta\omega\left(t-\frac{kT_0}{n-m}\right)\right]}{\mathrm{i}(n-m)\Delta\omega\left(t-\frac{kT_0}{n-m}\right)}.\tag{2}$$

In Eq. (2) (m_i, n_i) and $k=0, 1, \cdots N_t$, $N_t=(n-m)-1$ denote the scale and translation indices, respectively; and i is the subscript for the i-th scale. In the following, a uniform constant bandwidth is chosen for all scales under consideration, i.e., $n_i-m_i=n_j-m_j=n-m$, $i, j=1, 2, \cdots, N_\Omega$, $N_\Omega=N/2(n-m)$. Further, $T_0=\Delta t\cdot N$ is the time duration of the discretized signal, where N is the total number of sampling points; and $\Delta\omega=2\pi/T_0$.

Next, taking into account Eq. (2) and noticing that $\psi^{G}_{(m_i,n_i),k}(t)(n-m)\Delta\omega=\int_{m_i\Delta\omega}^{n_i\Delta\omega}e^{\mathrm{i}\omega\left(t-\frac{kT_0}{n-m}\right)}\mathrm{d}\omega$, the right hand side of Eq. (1) can be cast in the form

$$\sum_{p=-\infty}^{\infty}\psi^{G}_{(m_i,n_i),k}\left(t-pT_0\right)=\sum_{p=-\infty}^{\infty}\int_{m\Delta\omega}^{n\Delta\omega}\frac{1}{(n-m)\Delta\omega}e^{-\mathrm{i}\frac{\omega kT_0}{n-m}}e^{-\mathrm{i}pT_0\omega}e^{\mathrm{i}\omega t}\mathrm{d}\omega.\tag{3}$$

Further, considering the Poisson formula yields

$$\sum_{p=-\infty}^{\infty}\exp\left(-\mathrm{i}\omega pT_0\right)=\Delta\omega\sum_{q=-\infty}^{\infty}\delta(\omega-q\Delta\omega),\tag{4}$$

where $\delta(\cdot)$ is the Dirac delta function. Focusing on Eq. (3), changing the order of the summation and the integral operators, and taking into account Eqs. (1) and (4) yields an expression for the PGHW in the time domain of the form

$$\psi^{G,\mathrm{per}}_{(m_i,n_i),k}(t)=\frac{1}{n-m}\sum_{q=m_i}^{n_i}e^{\mathrm{i}\Delta\omega q\left(t-\frac{kT_0}{n-m}\right)}.\tag{5}$$

Obviously, applying the Fourier transform on Eq. (5) yields an expression for the PGHW in the frequency domain, i.e.,

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