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An efficient framework for the reliability-based design optimization of large-scale uncertain and stochastic linear systems

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ABSTRACT

This paper is focused on the development of an efficient reliability-based design optimization algorithm for solving problems posed on uncertain linear dynamic systems characterized by large design variable vectors and driven by non-stationary stochastic excitation. The interest in such problems lies in the desire to define a new generation of tools that can efficiently solve practical problems, such as the design of high-rise buildings in seismic zones, characterized by numerous free parameters in a rigorously probabilistic setting. To this end a novel decoupling approach is developed based on defining and solving a limited sequence of deterministic optimization sub-problems. In particular, each sub-problem is formulated from information pertaining to a single simulation carried out exclusively in the current design point. This characteristic drastically limits the number of simulations necessary to find a solution to the original problem while making the proposed approach practically insensitive to the size of the design variable vector. To demonstrate the efficiency and strong convergence properties of the proposed approach, the structural system of a high-rise building defined by over three hundred free parameters is optimized under non-stationary stochastic earthquake excitation.

1. Introduction

The benefits that can be achieved in terms of both performance as well as cost reduction through the application of numerical optimization to engineering problems are well known. In order to apply these methods to the design of optimal structural systems subject to environmental loads such as wind and earthquakes, the inherently dynamic and aleatory nature of the system and loads must be rigorously modeled. Indeed, it is well known that there is considerable uncertainty not only in the external environmental excitation but also in the parameters and models describing the system [1]. Recently, considerable effort has been placed on defining reliability-based/robust optimization approaches that describe the performance of the system in a rigorously probabilistic and dynamic setting, e.g. [1–4], see also [2] for a review. The recent interest in defining these approaches is a direct consequence of the latest computational advances that have opened the door to the possibility of solving problems that only recently would have been considered intractable. A hurdle that has remained a challenge is the possibility of solving reliability-based design optimization (RBDO) problems that are characterized by large design variable vectors. Indeed, of the methods so far developed, very few have considered problems with more than a handful of free design parameters [2]. This can be an important limitation as many practical applications are characterized by large design variable vectors, e.g. the design of typical multistory building systems. The difficulty in efficiently solving reliability-based design optimization, or robust optimization problems, with large design variable vectors is primarily due to how this limits the possibility of efficiently exploring how the probabilistic performance functions vary as the design variable vector changes during the optimization loop. Indeed, if the design variable vector has high dimensions, then surrogate/metamodel-based approaches [2] tend to become intractable as the exploration of the design space necessary to build the surrogate/metamodel will require a prohibitively large number of probabilistic performance evaluations. A similar curse of dimensionality also affects methods based on augmenting the uncertain vector with the design variables, as it becomes increasingly difficult to identify the regions of the design space that contain the optimal solutions. Another approach that has been

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widely adopted for solving RBDO problems is that based on decoupling the inherently nested probabilistic analysis from the optimization loop [5–8,1,2]. In general, this can only be approximately achieved and requires the sequential application of probabilistic analysis followed by the resolution of an approximate optimization sub-problem. The crucial point in these approaches is the construction of the sub-problem that generally requires additional information to be gathered on the local behavior of the performance functions around the current design point, e.g. through local random exploration or sensitivities. It is this phase that generally becomes troublesome as the design variable vector increases in size. In this paper a novel decoupling approach is developed that is practically insensitive to the size of the design variable vector. In particular, the methodology is specifically developed for high-dimensional uncertain linear dynamic systems driven by stochastic excitation. As an example of such a system, a case study is considered that focuses on the optimum design of the structural system of a high-rise building subject to non-stationary stochastic earthquake excitation.

2. Formulation of the optimization problem

The optimization problems that are pertinent to this research may be posed in the following form:

$$\text{Find} \quad \mathbf{x} = \{x_1, \ldots, x_m\}^T \tag{1}$$

$$\text{to minimize} \quad W = f(\mathbf{x}) \tag{2}$$

$$\text{s. t.} \quad P_{f_j}(\mathbf{x}) \le P_{0_j} \quad j = 1, \ldots, N_c \tag{3}$$

$$x_i \in \mathbf{x}_i \quad i = 1, \ldots, m \tag{4}$$

where $\mathbf{x}$ is an m-dimensional vector of deterministic parameters defining the design of the system, e.g. the section sizes, W is a deterministic and explicit (in terms of the design variable vector $\mathbf{x}$) cost function associated with the structural system, P_{f_j} are the failure probabilities associated with the N_c reliability constraints defining the performance of the system, P_{0_j} are the acceptable failure probabilities defining the target reliability of the system, while $\mathbf{x}_i$ is the discrete set to which the ith design variable must belong.

What makes the above outlined RBDO problem difficult to solve are the reliability constraints of Eq. (3). Indeed, the evaluation of the aforementioned constraints requires the calculation of the failure probabilities P_{f_j} given by

$$P_{f_j}(\mathbf{x}) = \int_{\Omega_{F_j}(\mathbf{x})} p(\mathbf{u})\, d\mathbf{u} \tag{5}$$

where Ω_{F_j} is the failure domain of the failure event F_j within the space of the uncertain parameters collected in the vector $\mathbf{U}$ (with $\mathbf{U}$ indicating the random vector and $\mathbf{u}$ a realization), while $p(\mathbf{u})$ is the joint probability density function of $\mathbf{U}$. In this work the random vector $\mathbf{U}$ describes all uncertainties involved in the system (model and loading parameters). In other words the components of the vector $\mathbf{U}$ represent the uncertain structural parameters and the random variables used in the characterization of the stochastic excitation. Therefore $\mathbf{U}$ will have high dimensions (order of thousands) which excludes the possibility of using analytical approximations, such as first and second order reliability methods, in the calculation of the probabilistic integral of Eq. (5) as they will become computationally intractable [9]. This implies that Eq. (5) must be evaluated using simulation methods and therefore through repeated evaluation of the system response. For practical dynamic systems, this fast becomes computationally cumbersome, especially when it is observed that P_{f_j} is in general an implicit function of $\mathbf{x}$ therefore hindering the calculation of the gradients necessary if efficient gradient-based optimization methods are to be used to solve the optimization problem.

3. Response estimation

3.1. Damage model

The failure events, F_j, of interest to this work may be written in the following form:

$$F_j(\mathbf{x}, \mathbf{u}) = \left\{ d_j(\mathbf{u}, \mathbf{x}) > 1 \right\} \tag{6}$$

where d_j is the damage measure associated with the jth reliability constraint and defined as follows:

$$d_j(\mathbf{u}, \mathbf{x}) = \max_{t \in [0,T]} \frac{|R_j(t; \mathbf{u}, \mathbf{x})|}{C_j} \tag{7}$$

where T is the duration of the event, $R_j(t)$ is the structural response process associated with the jth failure mode while C_j is a measure of the capacity of the system in the jth failure mode. In particular, C_j is directly related to the concept of fragility as defined in [10], i.e. as the conditional probability of having a predetermined damage state, DS_j, given a certain response level r. Indeed, if the capacity C_j is measured in terms of the response thresholds at which the damage state associated with the jth reliability constraint occurs, then the following holds:

$$\text{Fragility}_{DS_j} = P(DS_j|r_j) = P(C_j \le c_j) \tag{8}$$

where $r_j = c_j$. Therefore the distribution of C_j is simply given by the fragility curve associated with the damage state of interest. The relation of Eq. (8) illustrates how the damage ratio of Eq. (7), and in particular C_j, can be modeled using the extensive fragility databases reported in [11] for a number of common structural and non-structural building components.

By using the damage model of Eq. (7), a predefined damage state will occur if d_j is larger than 1. Therefore the following limit sate function can be assumed for identifying the initiation of damage:

$$g_j(\mathbf{u}, \mathbf{x}) = 1 - d_j(\mathbf{u}, \mathbf{x}) \tag{9}$$

while the failure probabilities are given by

$$P_{f_j}(\mathbf{x}) = P(g_j(\mathbf{u}, \mathbf{x}) \le 0) = \int_{g_j(\mathbf{u},\mathbf{x}) \le 0} p(\mathbf{u})\, d\mathbf{u} \tag{10}$$

In order to evaluate this integral, the response process $R_j(t)$ needs to be evaluated.

3.2. Load-effect model

In order to model the dynamic response of the system in a generic response parameter R_j (e.g. displacement, interstory drift, stress component), the following linear load-effect model is considered:

$$R_j(t) = s_1 \boldsymbol{\Gamma}_{R_j}^T \mathbf{K} \boldsymbol{\Phi}_n \mathbf{q}_n(t) \tag{11}$$

where s_1 is an uncertain parameter modeling the epistemic uncertainties in using a load-effect model of this type, $\boldsymbol{\Gamma}_{R_j}$ is a vector of influence coefficients indicating the response in R_j due to a unit static force applied one-by-one to the various degrees of freedom