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Transformation of a wave energy spectrum from encounter to absolute domain when observing from an advancing ship

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ABSTRACT

The article presents a practical approach to transform a wave energy spectrum from encounter domain to absolute domain. This problem has its specific relevance, when shipboard sea state estimation is conducted by the wave buoy analogy; notably for some particular implementation solving for the sea state directly in the encounter domain. In this context, the *encounter domain* is that observed from a ship when it advances in a seaway, whereas the *absolute domain* is that corresponding to making observations from a fixed point in the inertial frame. Spectrum transformation can be uniquely carried out if the ship sails “against” the waves (beam to head sea) but in following sea conditions there exists no unique solution to the problem. Instead, a reasonable approach valid for practical engineering must be applied, and the article outlines one viable solution that can be used to transform a wave spectrum from encounter to absolute domain. Specifically, two pseudo algorithms are presented, and good performance is achieved with both algorithms when they are tested at different operational scenarios.

1. Introduction

Today, marine vessels are heavily equipped with measuring sensors, and vast amount of information can be extracted by analysing the associated time history recordings. Typical 'observing sensors' are motion response units (MRUs), hull girder measuring devices (like strain gauges), and sea state recorders. Making a note on the latter, one means of a sea state recorder on a ship is given in terms of the *wave buoy analogy* [1]. With this analogy, sensor measurements of ship motions, or other global ship responses such as hull girder stresses, can be used to infer about the on-site sea state, in a similar way as is done with traditional floating wave buoys. Often, a vessel's measuring system and its sensors are part of an on-board decision support tool used to provide advice on speed and relative wave heading to the vessel's crew, and/or the monitoring system may be a component of larger shore-based measuring campaign applied to aid in strategic and business-related longer-term decisions.

Analyses of the measured wave-induced motions and other types of (structural) responses of a ship in a seaway can be made with few complications if the objective is merely to calculate statistics of the past measurements; usually carrying out the analyses by spectral analysis. Similarly, any observation of the wave system can be easily processed and analysed if interest concerns only the *encountered* wave energy distribution, equivalently wave spectrum. Here, it is understood that the 'encounter domain' is that one observed from the advancing ship, and it is assumed that advance speed and heading relative to the wave system do not change while observing. However, if real-time operator guidance with respect to (optimum) vessel speed and wave heading is in study, there is a need to transform the encountered wave energy spectrum to the (“true”) *absolute* domain; which is the domain any fixed observer without advance speed, relative to the inertial frame, is in. Thus, if not this transformation to absolute domain is made, it will not be possible to evaluate the effect of heading and/or speed changes of the vessel. The fundamental mathematics describing the problem – transforming from the encounter domain to the absolute domain, or vice versa – is basically governed by the Doppler Shift together with a requirement imposing energy conservation. If spectral analysis is carried out, the conservation of energy implies that corresponding sets of frequencies in the two domains map identical amounts of energy, as illustrated in Fig. 1.

From a theoretical point-of-view, the transformation problem is elementary, and most standard textbooks on naval architecture [2–5] consider the topic of spectrum transformation for a ship advancing forward with constant speed and at fixed heading relative to the waves that may be travelling in any direction.

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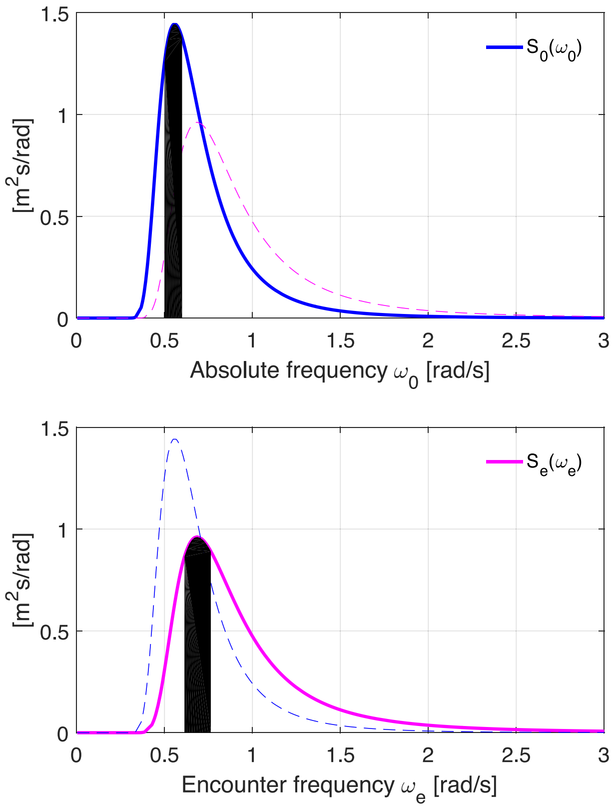

Fig. 1. Energy is conserved for corresponding set of frequencies in the absolute domain and the encounter domain for an observer moving at 10 knots against the waves at a 30° angle to the incoming waves. The distribution of wave energy follows a Bretschneider (Eq. (23)) wave spectrum with significant wave height H_s = 3.0 m, and zero-upcrossing period T_z = 8.0 s.

In practice, on the other hand, the problem is not easy to correctly address. Thorough and worthwhile readings on the topic are given by [6,7], where notably the latter points out the practical 'considerable complication' involved when transforming a (wave) spectrum. Nonetheless, both references have some detailed discussions and contain sort of "recipes" on how to transform a spectrum from the absolute domain to the encounter domain, but, disappointingly, they do not consider the reverse transformation – from encounter domain to absolute domain – and neither does other literature, available in English, seem to practically address the problem. It should, however, be pointed out[1] that attempts have been made by Japanese researchers in the past to secure 'frequency conversions in the evaluation of wave spectra' [8] but, unfortunately, details of the work are accessible only in Japanese. Later, an additional passing remark is given about [8].

The present article provides one possible solution, given in terms of a pseudo algorithm, which assures that the – from an advancing ship – observed wave energy spectrum is properly transformed from encounter domain to absolute domain. In the broader context, this problem and the corresponding solution have their specific interest, or their origin and necessity, when shipboard sea state estimation (SSE) is conducted by the wave buoy analogy; notably for some particular implementations of the wave buoy analogy when it is applied and solved for directly in the encounter domain using spectral analysis to carry out the fundamental (and initial) response analysis. For instance, this is the case with over-the-bow-looking wave recorders that basically combine sensor readings from a relative motion sensor and an accelerometer to provide the encountered wave elevation record that, subsequently, can be used to derive the single-point-wave spectrum [9,10]. The particular spectrum applies to the encounter domain and, as the final step and taking the relative heading between waves and vessel to be known, it is therefore necessary to transform to the absolute domain. More recently, the interest in a consistent transformation procedure has occurred due to research in a new implementation of the wave buoy analogy. The initial work [11] has considered vessels without advance speed, as focus was on ships being dynamically positioned. In this case, 'encounter domain' is equivalent to absolute domain and, hence, transformation is unnecessary, but a generalisation of the implementation, [12], for forward-moving ships requires indeed the estimated spectrum to be transformed from encounter to absolute domain. Moreover, newer but still quite conceptual studies, see [1], will also necessitate a way to carry out consistently the transformation of the wave energy spectrum.

The suggested transformation-procedure applies specifically to problems formulated through spectral analysis and concerns transformation in the one "direction" only; **from encounter domain to absolute domain**. This choice is made because of the specific context mentioned above. Apart from that, transformation in the other direction can be uniquely carried out [6,7] although, when the ship advances in following waves, generally the calculated encounter-wave spectrum can be heavily distorted, since three absolute-wave components map simultaneously into one encounter-wave component for some conditions. However, in the transformation from encounter to absolute domain there exists no unique solution on how to distribute the energy of an encounter-wave component to the three corresponding absolute-wave components, when the ship advances in following waves. These considerations should become more clear after reading the article. It is noteworthy that the presented solutions and the associated algorithms have focus on long-crested waves only, but this restriction can be relaxed by simply multiplying a parameterised directional spreading function ($\cos^{2s}$-type) on the transformed spectrum. Moreover, deep-water conditions are assumed throughout.

1.1. Composition of the article

The paper is organised into 6 sections. Following the Introduction, Section 2 lays out the theoretical formulations related to spectrum transformation and the section brings forward the practical complications. In Section 3, one specific solution is presented in terms of a (pseudo) algorithm to be applied when a wave energy spectrum needs to be transformed from the encounter domain to the absolute domain. Section 4 introduces a number of case studies upon which the performance of the transformation algorithm(s) can be tested, and the associated results and discussions are contained in Section 5. Finally, Section 6 puts down the conclusions from the study.

2. Ocean wave spectrum transformations

The transformation of wave energy spectra from the one domain to the other relies fundamentally on the fact that energy, totally, must be conserved. Thus, the governing theoretical formulation reads

$$S_e(\omega_e)d\omega_e = S_0(\omega_0)d\omega_0 \quad (1)$$

where ω_e and ω_0 are corresponding pairs of encounter frequencies and absolute wave frequencies, respectively, while S_e is the wave spectral ordinate in the encounter domain whereas S_0 is the spectral ordinate in the absolute domain. The energy equivalence, expressed by Eq. (1), was illustrated in Fig. 1, where a Bretschneider wave spectrum (significant wave height H_s = 3.0 m, zero-upcrossing period T_z = 8.0 s) was transformed from the absolute domain to

[1] Thanks to an anonymous reviewer.