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Vibration of two elastically mounted cylinders of different diameters in oscillatory flow

Toni Pearcey, Ming Zhao*, Yang Xiang, Mingming Liu

School of Computing Engineering and Mathematics, Western Sydney University, Penrith, NSW 2751, Australia

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ABSTRACT

Vibration of two elastically mounted cylinders in an oscillatory flow at a Keulegan-Carpenter number of 10 is simulated numerically. The two cylinders are rigidly connected with each other and are allowed to vibrate in the cross-flow direction only. The aim of this paper is to identify the effects of the orientation of the cylinders and the gap between the cylinders on the vibration. The two-dimensional Reynolds-Averaged Navier-Stokes equations are solved to predict the flow and the cylinder vibration is predicted using the equation of motion. When the two cylinders are in a tandem arrangement, a combined single pair flow regime and attached pair flow regime are observed as reduced velocity exceeds 10 and this combined regime and the single pair regime occurs intermittently. Periodic vibration is found when the two cylinders are in a staggered arrangement with a 45° flow attack angle. When the two cylinders are in a side-by-side arrangement, a new single vortex regime is observed. This single vortex remains attached to the cylinder surface and rotates around the cylinder. The intermittent switch between this single vortex regime and the single pair regime are observed.

1. Introduction

With increasing demand for energy resources from ocean including offshore oil and gas, renewable energy from wind, tidal currents and waves, more and more offshore structures are constructed. Offshore structures for extracting energy from ocean have to survive severe storms without damaging their functionality. Many cylindrical structures are used in offshore engineering such as subsea pipelines, risers, mooring cables, etc. Oscillatory flow is often used to model the water motion due to waves when the impact of the waves on small scale cylindrical structures is studied.

Many studies have been performed to understand the hydrodynamics and flow patterns around circular cylinders in oscillatory flows. It has been found that the hydrodynamic forces on cylindrical structures are mainly affected by the Keulegan-Carpenter (*KC*) number and the Reynolds number. The *KC* number is defined as $KC=(U_\mathrm{m}T)/D$, where U_m and T are the velocity amplitude and period of the oscillatory flow, respectively, and D is the diameter of the cylinder. The Reynolds number is defined as $\mathrm{Re}=U_mD/\nu$, where ν is the kinematic viscosity of the fluid. The ratio of the *KC* number to the Reynolds number is called the viscous parameter, β [15]. [21] conducted an experimental study of oscillatory flow past a circular cylinder for *KC* numbers ranging from 1 to 40 and classified the vortex flow into different flow regimes: Paring of attached vortices (non-vortex shedding regime) when $KC<7$, single pair regime when $7<KC<15$, double pair regime when $15<KC<24$, three-pair regime when $24<KC<32$ and four–pair regime when $32<KC<40$. Obasaju et al. [13] conducted a detailed study of the relationship between the vortex shedding regime and the hydrodynamic forces on a circular cylinder in an oscillatory flow. It was found that the spanwise correlation of the flow is good when *KC* is at the center of a regime and poor when *KC* is at the boundary between two regimes.

Numerical studies have been successfully conducted to investigate oscillatory flow past a circular cylinder. Some studies are mainly focused on the inception of the three-dimensionality of flow at low Reynolds numbers and low *KC* numbers [3,1,17,4]. Recently, research has been performed to study flow induced vibration (FIV) of circular cylinders in oscillatory flows. In addition to the Reynolds number and the *KC* number, FIV of a cylinder in oscillatory flow is also dependent on the mass ratio and the reduced velocity. The mass ratio is defined as $m^*=m/m_d$, where m is the cylinder mass and m_d is the displaced fluid mass, and the reduced velocity is defined as $V_r=U_m/(f_nD)$, where f_n is the structural natural frequency measured in vacuum in this study and many numerical studies. In many experimental studies of FIV in water, the natural frequency measured in still water (defined as f_nw in this study) is used to define

* Corresponding author.
E-mail address: m.zhao@westernsydney.edu.au (M. Zhao).

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Nomenclature

A_{max}	Maximum displacement
C_L	Lift coefficient
d	Small cylinder diameter
D	Large cylinder diameter
f_{nw}	Natural structural frequency measured in water
F_y	Lift force
f_n	Structural natural frequency
f_w	Oscillatory flow frequency
G	Gap between two cylinders
KC	Keulegan-Carpenter number
k_T	Turbulent kinetic energy
m	Cylinder mass
m^*	Mass ratio
m_d	Displaced fluid mass
p	Pressure
Re	Reynolds number
S_{ij}	Strain-rate tensor
S_y	Displacement of the mesh
T	Oscillatory flow period
U_m	Velocity amplitude
u	Sinusoidal velocity
u_i	Velocity component in the x_i-direction
$\hat{u}_i$	Mesh velocity
$\overline{u'_i u'_j}$	Reynolds stress tensor
V_r	Reduced velocity
x_i	The ith Cartesian coordinate
Y	Displacement of the cylinders
$\dot{Y}$	Velocity of the cylinders
$\ddot{Y}$	Acceleration of the cylinders
α	Flow attack angle
β	Viscous parameter
ν	Kinematic viscosity
ρ	Fluid density
ω	Vorticity
ω_f	Angular frequency of the flow
ω_T	Specific dissipation of turbulence

the reduced velocity. Experimental studies of the vibration of an elastically mounted circular cylinder in oscillatory flow show that the vibration of the cylinder locks in with a frequency which is a multiple of the oscillatory flow frequency [18,9].

Two pipelines of different diameters are often bundled together in offshore engineering to form a so-called piggyback pipeline. The piggyback pipeline configuration ensures the stability of the small-diameter pipeline. Many studies have been performed to investigate two cylinders of different diameters in fluid flow [11,10,7,20,8]. The attachment of the shear layer from the gap to the back surface of the larger cylinder was observed when two cylinders of different diameters are in a side-by-side arrangement in a steady flow [20,19]. Rahmanian et al. [14] studied vortex induced-vibration of two cylinders of different diameters at Re = 8000 and found that the vibration of two cylinders is significantly different from that of a single cylinder.

FIV of two cylinders of different diameters in an oscillatory flow, which is relevant to piggyback pipelines in waves in offshore engineering, has never been studied. In this study, FIV of two cylinders of different diameters as sketched in Fig. 1 (a) is studied numerically. The cylinders are allowed to vibrate only in the cross-flow direction. The diameters of the large and small cylinders are represented by D and d, respectively. The gap between the two cylinders and the flow attack angle is defined as G and α, respectively. The main objective of this study is to understand the effect of the position of the small cylinder relative to the large cylinder on the FIV. Simulations are conducted for a constant diameter ratio of $d/D = 0.2$, a constant mass ratio of 2, a constant Reynolds number (based on D) of 2×10^4, a constant KC number of 10, three gap ratios of $G/D = 0$, 0.1 and 0.2 and reduced velocities in a range of 1–20 with an interval of 1. A two-dimensional numerical model based on the RANS equations is used considering the large parametric space used in this study. A constant KC number of 10 is used, as the flow at this KC number presents the typical one pair flow regimes which occurs when $7 < KC < 15$. This regime is the KC number range that piggyback pipelines generally experience in ocean engineering.

2. Numerical method

The two-dimensional incompressible Reynolds-Averaged Navier Stokes (RANS) equations are used as the governing equations for simulating the flow. To account for the moving boundaries of the two cylinders, the Arbitrary Langrangian-Eulerian (ALE) scheme is applied to solve the RANS equations. The ALE scheme can avoid large deformations of the computational mesh because it allows the mesh to move with a velocity different from the fluid velocity. The effects of the mesh movement are considered by modifying the convection terms of the RANS equations. A Cartesian coordinate system is defined with its origin located at the center of the large cylinder and its x-coordinate pointing in the flow direction. The RANS equations in the ALE method are written as

$$\frac{\partial u_i}{\partial t} + \left(u_j - \hat{u}_j\right)\frac{\partial u_i}{\partial x_j} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_i}\left(2\nu S_{ij} - \overline{u'_i u'_j}\right) = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2)$$

where $x_1 = x$ and $x_2 = y$ are the Cartesian coordinates, u_i is the fluid velocity in the x_i direction, $\hat{u}_j$ is the mesh velocity, ρ is the fluid density, p is the pressure, S_{ij} is the strain-rate tensor and $\overline{u'_i u'_j}$ is the Reynolds stress tensor. The shear stress transport (SST) k-ω turbulence model developed by Menter [12] is used to close the RANS equations.

The equation of motion is used to calculate the displacement of the cylinder. For a one-degree-of-freedom problem, the equation of motion is:

$$m\ddot{Y} + c\dot{Y} + kY = F_y \quad (3)$$

where m is the total mass of the two cylinders, c is the damping coefficient of the system and k is the spring constant, Y, $\dot{Y}$ and $\ddot{Y}$ are the displacement, velocity and acceleration of the cylinders, respectively, F_y is the total hydrodynamic force of both cylinders in the cross-flow direction. The forces are found by integrating the shear stress and pressure over the cylinder surfaces.

The RANS equations are solved by the Petrov-Galerkin Finite Element Method (PG-FEM) developed by Zhao et al. [24]. Initially the fluid velocity and pressure are zero in the whole fluid domain and the cylinders are at their static balance position, i.e. Y and $\dot{Y}$ are zero. The boundary conditions are described as follows. At the left and right boundaries, the velocity is given as a sinusoidal flow as $u = U_m \sin(\omega_f t)$, where $\omega_f = 2\pi/T$ is the angular frequency of the flow and the pressure is given based on the undisturbed flow condition as $p = -\rho x \omega_f U_m \cos(\omega_f t)$. The turbulent quantities at the left and right boundaries of the computational domain are the specific dissipation turbulence $\omega_T = 1\ s^{-1}$ and turbulent kinetic energy $k_T = 0.001\omega_T^2 2$. On the two side boundaries that are parallel to the flow direction, the velocity perpendicular to the boundary is zero and the gradient of all other quantities in the normal direction of the boundary are zero. On the cylinder surfaces, no-slip boundary condition is used, i.e. the velocity of the fluid is the same as the