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A practical method for combination of fatigue damage subjected to low–frequency and high–frequency Gaussian random processes

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ABSTRACT

This paper addresses the combination of fatigue damage for offshore steel structures subjected to low–frequency and high–frequency Gaussian components. Two new formulae (Eq. (25) and Eq. (38)) are developed. To verify the accuracy of the two formulae, extensive numerical simulations on bimodal spectra are carried out, and the results calculated by the two new formulae are satisfactory. In particular, compared with other methods (DNV methods, Huang–Moan method, Temple method, Combined spectrum approach, Jiao–Moan method, Low's bimodal method and Low–2014 method), one of two formulae (Eq. (38) in this paper) is accurate and simple enough and valid for a wide range, which shows a strong attraction for application in engineering design.

1. Introduction

Fatigue is an important design criterion for offshore steel structures subjected to various types of Gaussian random processes, in which one special case is the combination of a high–frequency (HF) Gaussian component and a low–frequency (LF) Gaussian component. A typical example is the wave frequency (WF) and low frequency (LF) drift responses in the risers and mooring systems of floating offshore installations, in which the response spectra exhibit two modes for the different loads [1].

In general, the methods for evaluating fatigue damage under combination of LF and HF Gaussian random processes can be placed into two categories: the time domain method and the frequency domain method. When a stress history can be easily obtained, the time domain method is usually adopted for estimating the fatigue life of structural and mechanical components, based on the rainflow cycle counting technique [2] and a linear damage accumulation rule (Palmgren and Miner rule) [3]. The fatigue damage obtained by the time domain method is regarded as the most accurate solution. The time domain method is hence used in the detail design stage of structures to ensure safety and reliability. However, to obtain stable fatigue damage results, many time series samples must be provided, which could be extremely time-consuming. On the contrary, frequency-domain methods are widely used as efficient and time-saving techniques and may be the most suitable tools to apply in every stage of design.

Several spectral methods in bimodal processes are developed for estimating fatigue damage—e.g., combined spectrum approach [4], Jiao–Moan method [5], Sakai–Okamura method [6], Fu–Cebon method [7], Low's bimodal method [8] and Low–2014 method [9]. The first method is simple but conservative and has been applied in DNV-OS-E301 [4]. Jiao–Moan method, Sakai–Okamura method and Fu–Cebon method are compared by Benasciutti and Tovo [10], who concluded that the Jiao–Moan method gives rather exact results in a wide range. More importantly, the Jiao–Moan method is also widely used in offshore rules, such as DNV-OS-E301 [4], DNV-OS-F201 [11], DNV-RP-F204 [1], API-RP-2SK [12] and ISO-19901-7 [13]. Low's bimodal method gave a new damage prediction model which greatly reduced the conservation of fatigue damage due to the large cycles and small cycles. Low–2014 developed a simple formula for evaluating the fatigue damage in bimodal random processes, which is calibrated by numerical simulations. These two methods provided a great improvement in accuracy of fatigue damage.

In some cases of structural design, compared with the bimodal spectral methods previously mentioned, it is more convenient to calculate the contribution of fatigue damage due to LF and HF random processes separately. An instance of such a design situation is swell response of an FPSO that is also subjected to a wave response [14]. Furthermore, for the fatigue design of offshore wind turbines, it is practical to evaluate fatigue damage by separating the wind response and wave response. Since the design of turbine and the support structures belong to different contractors, which lead to a

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situation that wind–induced fatigue is calculated in time domain, wave–induced fatigue is computed in frequency domain [15]. For these two cases, the total fatigue damage due to two dynamic processes analyzed separately should be calculated through combined fatigue damage methods.

A systematic investigation is found in the literature for the combination of fatigue damage. It is firstly recommended to evaluate the combined fatigue damage by a simplified DNV method that has been widely applied in DNV specifications such as DNV-RP-F204 [1], DNV-OS-F201 [11] and DNV-RP-C203 [14]. This approach was originally proposed by Lotsberg [16] and based on the information of the mean zero up-crossing frequency and the individual damage. However, it yields a highly conservative result of the combined fatigue damage with a derivation of as much as 200% [17]. Another simple method for combination of fatigue damage was proposed by Huang and Moan [17], and its expression is analogous to the simplified DNV method. In fact, it is a correction to the narrow band approximation through an irregular factor. In addition, a formula proposed by Temple [15] for combining the fatigue damage is adopted, and equals the square root of the quadratic summation of the individual damage. This superposition solution of damage has been applied in the Utgrunden wind farm [15] for evaluating fatigue damage due to wave and wind loads. Unfortunately, this method lacks theoretical background, and its accuracy has not been verified. Therefore, it is essential to establish a simple, practical and precise formula for a combination of fatigue damage subjected to LF and HF Gaussian random processes.

The purpose of this paper is to develop an engineering method and give specific analytical formulae that can be used for combining fatigue damage under LF and HF Gaussian random processes. Numerical simulations are conducted to validate the accuracy of the proposed methods. Moreover, the existing methods for combining fatigue damage and bimodal spectra methods are also compared with the new methods.

2. Basic theory

2.1. Theory of fatigue analysis

The material resistance to fatigue failure can be described by an *S–N* curve defined by the form:

$$N = K \cdot S^{-m} \tag{1}$$

Where S represents the stress level (assumed herein to be the stress amplitude), N is the number of cycles to fatigue failure, and K and m are the fatigue strength coefficient and fatigue strength exponent, respectively, obtained from the constant amplitude fatigue test.

Fatigue damage is defined as the ratio between the number of cycles in the design lifetime and that to fatigue failure. The total fatigue damage can then be calculated as a linear accumulation rule after Palmgren and Miner:

$$D = \sum \frac{n_i}{N_i} = \frac{1}{K}\sum n_i S_i^m = \frac{n}{K}\overline{S^m} \tag{2}$$

where n_i is the number of cycles in the stress amplitude S_i, resulting from rainflow counting; N_i is the number of cycles corresponding to fatigue failure at the same stress amplitude, based on the *S–N* curve; n represents the total number of cycles in the duration; and $\overline{S^m}$ is the mathematical expectation of S^m

When the stress amplitude is a continuum function and its probability density function (PDF) is $f_S(S)$, $\overline{S^m}$ can be denoted as follows:

$$\overline{S^m} = \int_0^\infty S^m \cdot f_S(S)dS \tag{3}$$

2.2. Spectral characterization of random processes

For a stationary Gaussian random process $X(t)$ assumed herein to have a zero mean value, it can be expressed by a one-sided power spectral density $S_X(\omega)$ in the frequency domain. A set of spectral moments can be defined as:

$$\lambda_i = \int_0^\infty \omega^i \cdot S_X(\omega)d\omega \tag{4}$$

where subscript i can usually take an integral value, such as $i = 0$, 1, 2, 4, which corresponds to the i-th-order spectral moment.

Other typical parameters are the mean zero up-crossing frequency ν_0 and the peak occurrences frequency ν_p, which depend on spectral moments in frequency domain and can be determined from Eq. (5):

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}},\ \nu_p = \frac{1}{2\pi}\sqrt{\frac{\lambda_4}{\lambda_2}} \tag{5}$$

Moreover, bandwidth is also an important parameter that characterizes a random process. A group of bandwidth parameters is defined as follows:

$$\alpha_i = \frac{\lambda_i}{\sqrt{\lambda_0 \lambda_{2i}}} \tag{6}$$

The most commonly used bandwidth parameters are given as

$$\alpha_1 = \frac{\lambda_1}{\sqrt{\lambda_0\lambda_2}},\ \alpha_2 = \frac{\lambda_2}{\sqrt{\lambda_0\lambda_4}} \tag{7}$$

where α_2 is the irregular factor, representing the ratio between ν_0 and ν_p, and the range of the bandwidth parameters is $0 \le \alpha_1 \le 1$, $0 \le \alpha_2 \le 1$.

In Eq. (6), i can also take a non-integral value; for example, $\alpha_{0.75}$, $\alpha_{0.97}$ and $\alpha_{1.97}$ are used in the literature [18,19].

Two spectral parameters, relating to α_1 and α_2, are defined as follows [20,21]:

$$\delta = \sqrt{1-\alpha_1^2} = \sqrt{1-\frac{\lambda_1^2}{\lambda_0\lambda_2}},\ \varepsilon = \sqrt{1-\alpha_2^2} = \sqrt{1-\frac{\lambda_2^2}{\lambda_0\lambda_4}} \tag{8}$$

where δ is Vanmarcke's spectral parameter with $0 \le \delta \le 1$, and ε is Wirsching's spectral width parameter with $0 \le \varepsilon \le 1$.

For a Gaussian random process with zero mean, the PDF of peaks u follows the Rice distribution which has an analytical expression [22] as given in Eq. (9):

$$f_u(u) = \frac{\sqrt{1-\alpha_2^2}}{\sqrt{2\pi\lambda_0}}\exp\left(-\frac{u^2}{2\lambda_0(1-\alpha_2^2)}\right) + \frac{\alpha_2 u}{\lambda_0}\exp\left(-\frac{u^2}{2\lambda_0}\right)\Phi\left(\frac{\alpha_2 u}{\sqrt{\lambda_0}\sqrt{1-\alpha_2^2}}\right) \tag{9}$$

where $\Phi(\cdot)$ is the standard normal distribution function.

For an ideal narrow band process, the Rice distribution turns into the Rayleigh distribution and the amplitude distribution coincides with the peak distribution; thus, the analytical expression is given as

$$f_S(S) = \frac{S}{\lambda_0}\exp\left(-\frac{S^2}{2\lambda_0}\right) \tag{10}$$