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Analysis of wave groups by wave envelope-phase and the Hilbert Huang transform methods

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A B S T R A C T

The wave groups are studied by both conventional wave analysis methods and by the non-stationary Hilbert Huang Transform (HHT) method. Full-scale wave records containing abnormal waves are used. Instantaneous quantities, such as envelope, phase and frequency, are adopted to study the wave grouping. A refined definition of wave group is proposed considering that the wave process is simultaneously amplitude and frequency modulated. The validation of the proposed definition is conducted by analysis of numerical simulation data. Group parameters are proposed based on the time-frequency distribution of energy. An attempt is made to find the relationship between the characteristics of abnormal waves and the group characteristics.

1. Introduction

Wave groups are time-localized events that have been shown to occur more often than it would be expected if the waves are random. The occurrence of runs of successive high waves affects a wide range of ocean and coastal activities. Wave groups can pose hazards for fixed and floating structures, and they can lead to strong oscillations in harbours and bays (e.g. [2,23]). The transformation of the wave groups in the process of wave breaking is related to the generation of low frequency waves in the surf zone, edge waves, shear instability waves and fluctuation of rip currents (e.g. [1,15,21,31]).

The interest in the wave groups is also increased in connection with the study of abnormal waves. The evolution of wave groups, which can lead to wave focusing due to nonlinear interactions, is considered as one of the possible mechanisms explaining abnormal wave occurrences (e.g. [28,36]). It is related to the redistribution of the spectral energy due to wave nonlinear interactions that transform the local wave groups into one single large wave. The theory of the modulation instability presumes unidirectional narrow-banded waves in deep water, which were reproduced in a tank [27,30] or numerically simulated (e.g. [32,41,42]). Since the wave groups are considered a stage of the evolution process towards formation of abnormal wave, knowledge of group characteristics is required. This study aims to explore if there is any relation between the group characteristics of a wave record, containing abnormal wave and the characteristics of the abnormal wave itself.

Earlier the individual groups were identified by counting the number of discrete wave heights that exceed a prescribed height and this measure of wave groupiness was called a group run length [9]. Later the envelope function was adopted as a detector of wave groups as the wave groups are defined by consecutive crossings of a given threshold (e.g. [14,18,24]. The similarities and relationships among the various parameters and methodologies commonly used for wave group analysis are reviewed by [19] and [26]. In addition, more sophisticated analysis methods that deal with the local properties of waves have also been used to study the wave groupiness as in addition to the envelope function, the phase and the instantaneous frequency, are also adopted (e.g. [3,5,16]). Most of the studies are dedicated to the statistics and modelling of linear wave groups, but the work of [34] focused on the nonlinear wave groups showing that the nonlinearities do affect the statistics of large wave crests and their groups.

In this work, two approaches are used to study wave groups. First the sea wave groups are analysed by the conventional methods for wave data analysis. Additionally to the wave envelope, the instantaneous phase is adopted to take into account that the wave process is not only amplitude modulated but also the phase modulated one. The statistics of the envelope and phase, theoretically derived for narrow banded process, are used to redefine individual wave groups. The new wave group definition is validated by analysis of numerical simulation data. The modified JONSWAP

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spectrum with parameters determined from the observations is used for the linear random numerical simulations. Second, since the existence of wave groups causes parceling of wave energy, the time-frequency distribution of energy defined by the Hilbert-Huang Transform method is adopted here complimentary to the conventional methods for studying wave groups. The individual wave group is characterized not only by its time duration but also by its group energy, determined from the time-frequency distribution of energy. An attempt is made to find the relationship between the characteristics of abnormal waves and the group characteristics.

The paper is organized as follows: The methods of analysis are presented in Section 2 together with details about the numerical simulation. The data are briefly described in Section 3, while the results of the wave group analysis by the conventional and the HHT methods are presented in Section 4.

2. Methods for analysis

2.1. Envelope, phase and instantaneous frequency

Let η represents the sea surface elevation from the mean sea level, observed at a fixed point and at time t. The analytical function $\xi(t)$ defined as

$$\xi(t) = \eta(t) + j\hat{\eta}(t) \quad (1)$$

corresponds to the vertical displacement of sea surface elevation $\eta(t)$. Here $\hat{\eta}(t)$ is the Hilbert transform of sea surface elevation given by

$$\hat{\eta}(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\eta(t')}{t'-t} dt' \quad (2)$$

where P indicates the Cauchy principal value of the integral.

The envelope $A(t)$, the phase $\varphi(t)$ and the instantaneous frequency $\omega(t)$ of the sea surface elevation $\eta(t)$ are defined respectively as

$$A(t) = \sqrt{\eta^2(t) + \hat{\eta}^2(t)} \quad (3)$$

$$\varphi(t) = arctg\left[\frac{\hat{\eta}(t)}{\eta(t)}\right] \quad (4)$$

$$\omega(t) = \frac{d\varphi}{dt} \quad (5)$$

For narrow band Gaussian stationary process with zero mean, the phase $\varphi(t)$ can be considered decomposed into a linear part $\omega_0 t$ and a deviation part $\theta(t)$ as

$$\varphi(t) = \omega_0 t + \theta(t) \quad (6)$$

where ω_0 is a representative or carrier frequency.

The instantaneous frequency (5) then is

$$\omega(t) = \omega_0 + \frac{d\theta(t)}{dt} \quad (7)$$

The time derivative of phase $\theta' = d\theta/dt$ gives the deviation from the carrier frequency ω_0 of the narrow-band wave process. The positive (negative) values of θ' correspond to the situation where the local frequency is higher (lower) than the carrier frequency ω_0.

The conditional probability density of the envelope for a given value of the local frequency θ' is derived from the probability density function of envelope $A(t)$, phase $\theta(t)$ and their derivatives as

$$p\left(A|\theta'\right) = \frac{4}{\sqrt{\pi}} A^2 \left(\frac{\mu_2/m_0 + \theta'^2}{2\mu_2}\right)^{3/2} exp\left(-\frac{A^2\left(\mu_2/m_0 + \theta'^2\right)}{2\mu_2}\right) \quad (8)$$

where m_0 is the zero-th spectral moment and μ_2 is the central spectral moment of the sea surface elevation (e.g. [29]).

The expected value and variance of the conditional probability density function (8) are defined respectively as

$$E\left[A|\theta'\right] = \sqrt{\frac{8}{\pi}} \frac{\sigma_1}{\sqrt{\mu_2/m_0 + \theta'^2}} \quad (9)$$

$$D\left[A|\theta'\right] = \frac{(3\pi - 8)\sigma_1^2}{\pi\left(\mu_2/m_0 + \theta'^2\right)} \quad (10)$$

The mean (9) and variance (10) tend to zero for large values of the phase derivatives θ'. Thus for the case of narrow band wave process, the high frequency waves are expected to have small amplitudes.

For a given value of the local frequency θ', the conditional probability density function (8) has a maximum when

$$A_m = \sqrt{\frac{2\mu_2}{\mu_2/m_0 + \theta'^2}} \quad (11)$$

A_m is the most probable amplitude for a given value of local frequency, and it is connected with the phase modulation of wave process.

The envelope function (3) isolates the mean and the difference interaction terms, and it provides a direct measure of amplitude modulations that are occurring at all frequencies. In this work the envelope $A(t)$ is used as a detector of wave groups. If a wave group is defined by two consecutive crossings of a given threshold then the wave group is equivalent to the run of high waves [9]. Different wave characteristics such as mean wave height or significant wave height have been used as a threshold.

Additionally the phase $\theta(t)$ defined by (4) is also considered here. Fig. 1a presents an example of the sea surface elevation, shown by tin line and its envelope (thick line), while the corresponding phase $\theta(t)$ is shown in Fig. 1b. It is seen that the amplitude modulation superimposed over the groups (Fig. 1a) is accompanied with a phase modulation in Fig. 1b. The phase function $\theta(t)$ has negative slope during the wave groups and consequently the local frequency during the wave group is lower than the carrier frequency ω_0. A significant phase change is observed between the groups, where amplitudes are very small. Indeed according to (9), the expected value of envelope $E[A|\theta']$ for given frequency tends to zero for large values of the phase derivatives. Guedes Soares and Cherneva [11] reported similar results of analysis of wave groups by joint consideration of the envelope, the phase, the phase change rate and the spectrogram of wind waves. The importance of phase-modulated packets for the formation of abnormal waves is pointed by [20]. It was shown that the phase modulation of the initial nonlinear wave field leads to a significant intensification of the process of abnormal wave generation.

Fig. 2 illustrates the definition of wave group. The mean envelope A_{mean} is shown by dashed thick line. Taking into account the fact that the wave process is also phase modulated, a new thershold equal to the most probable amplitude A_m for given value of frequency (see Eq. (11)) is proposed. It is shown by dash-dotted line. The wave groups, defined by two thresholds, A_{mean} and A_m, are schematically presented by vertical lines in Fig. 2.

The statistics of linear wave groups were derived as a function of spectral bandwidth parameter ν for the case of narrow-banded process by [24]. Using a second-order representation for crest heights and the wave envelope approach [34] derived the statistics of nonlinear wave groups. The average number of waves H_y in a group with crest heights above threshold y is expressed as

$$H_y = \frac{\omega_{02}}{\omega_{01}\nu\sqrt{2\pi}} \frac{1}{r}, \quad as \quad y = r\left(1 + \frac{1}{2}\mu^* r\right) \quad (12)$$